

Applications: The Equity Term Structure and Other Recent Work

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Outline

Recent Facts and Puzzles

A GE Model for Risky Return Puzzles

A Workhorse Partial Equilibrium Model

Final Words

Background: Empirical Facts and Puzzles

Classic (“first generation”) asset-pricing facts:

1. Equity premium puzzle [Mehra & Prescott (1985), Hansen & Jagannathan (1991)]: High $\bar{\mu} - r_f$.
2. Risk-free rate puzzle [Weil (1989)]: Low and stable r_f . (Using high risk aversion to solve equity premium puzzle generates very high and volatile r_f .)
3. Equity volatility puzzle [Shiller (1981)]: Stock prices are much more volatile than dividends.

A small sampling of recent facts and puzzles:

- ▶ Low correlation puzzle [e.g., Barberis, Huang, Santos (2001)]: Low correlation between returns and consumption (or other fundamentals). (Maybe a first-gen puzzle.)
- ▶ Risky term structure [e.g., van Binsbergen & Koijen (2017), Gormsen & Lazarus (2022)]: While the risk-free yield curve is upward sloping on average, the term structure of returns on *dividend strips* — claims on one-period dividends at some horizon — is flat or downward sloping.
- ▶ Macroeconomic announcement premia [Savor & Wilson (2013), Lucca & Moench (2015)]: A large fraction of the overall equity premium is earned on trading days with macro announcements.
- ▶ Puzzles in other asset classes (FX, bonds, housing, ...), arbitrage violations (CIP, bond-CDS basis, ...).

Background: More on the Risky Term Structure

- ▶ Figure from Gormsen (2021) of average one-year returns to short-term dividend claim (“Short-maturity claims”) and the overall market (“Long-maturity claims”):

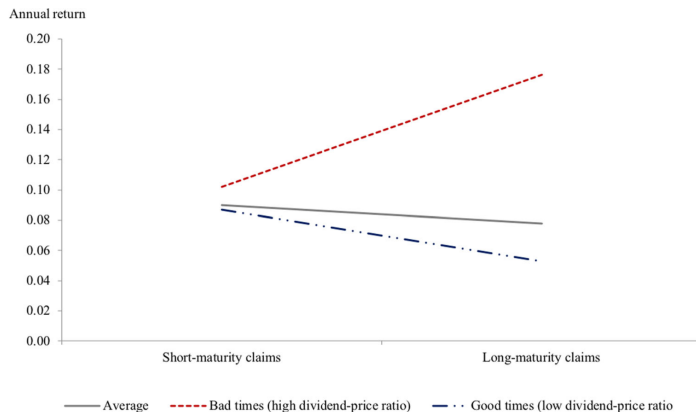
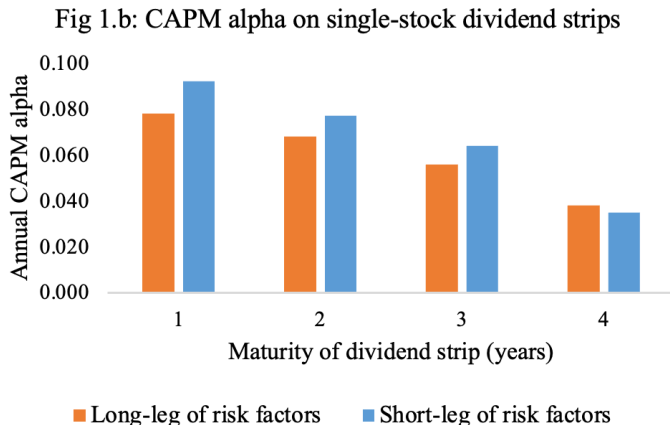


Figure 1. The term structure of one-year equity returns. This figure plots the term struc-

- ▶ Term structure of expected returns is roughly flat on average (also, oddly, countercyclical).

Background: More on the Risky Term Structure

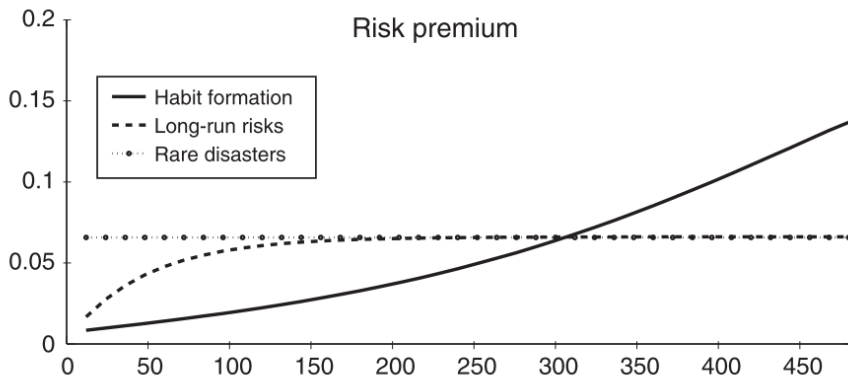
- ▶ Figure from Gormsen & Lazarus (2022) of average **risk-adjusted returns** (CAPM alphas) for **single-firm** dividend claims:



- ▶ Raw expected returns flat, but alphas robustly decreasing. (Use this to understand cross-section.)

Background: More on the Risky Term Structure

- Figure from van Binsbergen & Kojien (2012) shows how models written to explain first-gen puzzles miss this (x-axis is maturity in months):



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A GE Model for Risky Return Puzzles

- ▶ Atmaz and Basak (*JF*, 2022): Nice recent paper explaining downward-sloping equity term structure, low correlation puzzle, and related recent puzzles, using a GE model that distinguishes between dividend-paying and non-dividend-paying stocks.
- ▶ The basic insight is that non-dividend stocks have:
 - ▶ **No** contemporaneous correlation with consumption.
 - ▶ Longer-horizon cash flows (after uncertain dividend initiation date).
 - ▶ Very uncertain long-term fundamentals, since the lack of current dividends is a form of information incompleteness.
- ▶ The model is stylized, and non-dividend stocks aren't a big enough share of the market (about 20% in market cap terms) for this to be the whole story.
- ▶ I prefer interpreting "non-dividend stocks" somewhat more generally than the paper: can be thought of as growth firms with comparably poor info quality.

Model Setup: Stocks

- ▶ Two types of stocks. **Type 1** is dividend-paying stock, with dividends

$$\frac{dD_{1t}}{D_{1t}} = \mu_1 dt + \sigma_1 dZ_{1t},$$

where μ_1 and σ_1 are constant, and Z_{1t} is a Brownian motion.

- ▶ **Type 2** is non-dividend stock. It pays out dividends starting at random stopping time τ , and dividend dynamics are

$$\frac{dD_{2t}}{D_{2t}} = \mu_2 dt + \sigma_2 dZ_{2t},$$

with $dZ_{1t}dZ_{2t} = \rho dt$ (correlation ρ).

- ▶ Dividend initiation: Poisson with intensity $\lambda_2 \implies \tau \sim \text{Exponential}(\lambda_2)$, so $E_0[\tau] = 1/\lambda_2$.
- ▶ For $t < \tau$, D_2 is evolving unobservably according to the dynamics above (“pseudo-dividend”).
- ▶ Stock market value: $S_t = S_{1t} + S_{2t}$.
- ▶ Model is non-stationary: after τ , no interesting dynamics. (Published version of paper has a trick to generate stationarity.)

Model Setup: Information

- Prior to τ , investors can observe only D_1 directly. For D_2 , observe noisy fundamental news F_2 , which gives info about true future dividend process via cointegrating relationship:

$$d(f_{2t} - d_{2t}) = \kappa_2[\zeta_2 - (f_{2t} - d_{2t})]dt + v_2 dZ_{2t}^*,$$

where $f_{2t} = \log F_{2t}$, $d_{2t} = \log D_{2t}$, and Z_{2t}^* is independent of other Brownian motions. Think of F_2 as earnings, and over long run (rate $\kappa_2 > 0$) dividends revert to constant mult. of earnings.

- Prior: $d_{20} \sim \mathcal{N}(\hat{d}_{20}, V_{20})$. More Liptser & Shiryaev: Belief evolves as

$$\frac{d\hat{D}_{2t}}{\hat{D}_{2t}} = \mu_2 dt + \rho\sigma_2 dZ_{1t} + \frac{(1 - \rho^2)\sigma_2^2 + \kappa_2 V_{2t}}{\sqrt{1 - \rho^2\sigma_2^2 + v_2^2}} d\hat{Z}_{2t},$$
$$dV_{2t} = - \left[\frac{((1 - \rho^2)\sigma_2^2 + \kappa_2 V_{2t})^2}{(1 - \rho^2)\sigma_2^2 + v_2^2} - (1 - \rho^2)\sigma_2^2 \right] dt,$$

where \hat{Z}_{2t} is Brownian motion under investor's info set (independent of Z_{1t}). Estimated \hat{D}_2 is more volatile than D_2 .

Model Setup: Preferences and Endowment

- ▶ Rep. agent with CRRA preferences (rate of time preference β , risk aversion γ).
- ▶ Endowment dynamics:

$$\frac{dY_t}{Y_t} = \begin{cases} (\alpha_1 + \alpha_2)\mu_1 dt + (\alpha_1 + \alpha_2)\sigma_1 dZ_{1t}, & t < \tau, \\ (\alpha_1\mu_1 + \alpha_2\mu_2)dt + \alpha_1\sigma_1 dZ_{1t} + \alpha_2\sigma_2 dZ_{2t}, & t \geq \tau. \end{cases}$$

- ▶ For $t \geq \tau$, α_1 and α_2 are sensitivities of consumption to each dividend (e.g., could set $\alpha_1 = \alpha_2 = \frac{1}{2}$). Prior to this period, though, their modeling trick is to increase the sensitivity of cons. to stock 1 dividend. This leads to constant sensitivity of aggregate consumption to aggregate dividend.
- ▶ To close model, risk-free asset in zero net supply.

Equilibrium

- ▶ State-price density $\pi_t = e^{-\beta t} (Y_t/Y_0)^{-\gamma}$.
- ▶ In “normal” period, $t \geq \tau$, $\frac{d\pi_t}{\pi_t} = -\bar{r}dt - \gamma\alpha_1\sigma_1 dZ_{1t} - \gamma\alpha_2\sigma_2 dZ_{2t}$, so for $n = 1, 2$,

$$\frac{d\pi_t D_{nt}}{\pi_t D_{nt}} = -(\bar{r}_n - \mu_n)dt + (1 - \gamma\alpha_n)\sigma_n dZ_{nt} - \gamma\alpha_{-n}\sigma_{-n}dZ_{-nt},$$

where $\bar{r}_n = \bar{r} + \gamma(\alpha_n\sigma_n^2 + \alpha_{-n}\rho\sigma_n\sigma_{-n})$ is expected return for S_{nt} .

- ▶ Constant expected returns & growth gives Gordon growth model in this period: $S_{nt} = \frac{D_{nt}}{\bar{r}_n - \mu_n}$.
- ▶ In “main” period, $t < \tau$, dividend-paying stock price is

$$S_{1t} = E_t \left[\int_t^\tau e^{-\beta(s-t)} \frac{Y_s^{-\gamma}}{Y_t^{-\gamma}} D_{1s} ds + \frac{\pi_\tau}{\pi_t} S_{n\tau} \right],$$

with exponential τ generating additional discount λ_2 in first term.

- ▶ Using this with normal-period stock price solution, get price $S_{1t} = \frac{1}{\bar{r}_1 - \mu_1} \frac{\bar{r}_1 - \mu_1 + \lambda_2}{\bar{r}_1 - \mu_1 + \lambda_2} D_{1t}$, with $r_1 = r + \gamma(\alpha_1 + \alpha_2)\sigma_1^2$, for main period.

Price and Return for Non-Dividend Stock

- ▶ Non-dividend stock price is just equal to discounted expectation of price as of dividend initiation:

$$S_{2t} = E_t \left[\frac{\pi_\tau}{\pi_t} S_{2\tau} \right] = \frac{1}{\bar{r}_2 - \mu_2} E_t \left[\frac{\pi_\tau}{\pi_t} D_{2\tau} \right].$$

- ▶ Use learning dynamics, along with the fact that learning doesn't affect consumption or SPD in main period:

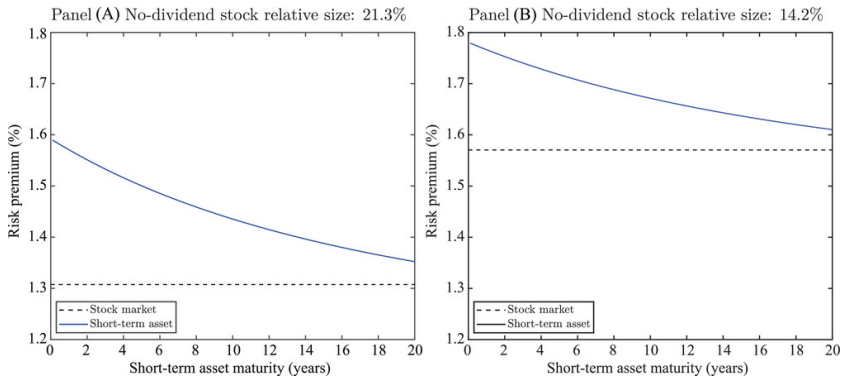
$$E_t [\pi_\tau D_{2\tau}] = E_t [\pi_\tau \hat{D}_{2\tau}] = E_t \left[\int_t^\infty \pi_s \hat{D}_{2s} \lambda_2 e^{-\lambda_2(s-t)} ds \right] = \frac{\lambda_2}{r_2 - \mu_2 + \lambda_2} \pi_t \hat{D}_{2t},$$

where $r_2 = r + \gamma(\alpha_1 + \alpha_2)\rho\sigma_1\sigma_2$ (stoch. component of \hat{D}_{2t} that covaries with π_t is $\rho\sigma_2 dZ_{1t}$).

- ▶ Consider $\sigma_1 = \sigma_2$. In main period, $r_2 - r = \gamma(\alpha_1 + \alpha_2)\rho\sigma_1^2 < \gamma(\alpha_1 + \alpha_2)\sigma_1^2 = \bar{r}_2 - \bar{r} \implies$ risk premium is **lower** for more uncertain stock, since it covaries with aggregate consumption only ρ for 1!
- ▶ Meanwhile, the opposite is true for the dividend stock: $r_1 - r > \bar{r}_1 - \bar{r}$.

The Aggregate Market and the Term Structure of Returns

- ▶ Assuming constant relative sizes of the two stocks in the two periods ($S_{nt}/S_t = \bar{S}_{nt}/\bar{S}_t$) and larger dividend-paying share than non-dividend share, then obtain a risk premium that decreases as of τ .
- ▶ Because of this, the market has a higher short-term than long-term expected return:



Consumption Correlations

- In addition, the presence of the non-dividend-paying stock helps address the low consumption correlation puzzle:

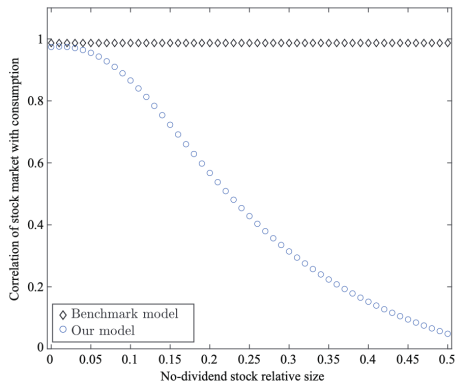


Figure 1. Correlation of stock market returns with consumption growth rate. This figure plots the equilibrium correlation of stock market returns with the aggregate consumption growth rate by varying the no-dividend relative stock size S_{it}/S_t in our economy (blue circles). The corresponding correlation in the benchmark economy in which all stocks pay dividends is obtained by setting the no-dividend stock relative size to zero (black diamonds). Parameter values are reported in Table I and are discussed in Section V. (Color figure can be viewed at wileyonlinelibrary.com)

Other Predictions

- ▶ Also generates predictions about the cross-section (no-dividend stock has higher beta but lower returns, exactly like growth or long-duration stocks) and relationship between volatility and returns.

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- ▶ Before wrapping up, want to put a word in for Lettau and Wachter (2007), which is an extremely tractable and useful partial equilibrium model that foresaw the second-generation AP puzzles.
- ▶ The model is partial equilibrium because the SDF is exogenously specified, like is often the case in affine term-structure models.
- ▶ Basically a standard term-structure model, but applied to the risky term structure. I use it all the time.
- ▶ Will discuss in discrete time (as in the paper), but worth thinking through how it maps to continuous time.

The Lettau-Wachter Model

- Aggregate log dividend growth:

$$\delta d_{t+1} = g + z_t + \sigma_d \varepsilon_{d,t+1},$$

$$z_{t+1} = \phi_z z_t + \sigma_z \varepsilon_{z,t+1},$$

with $\rho_{dz} = \text{Corr}(\varepsilon_{d,t+1}, \varepsilon_{z,t+1}) < 0$. This implies mean reversion: negative shocks to current dividends are partly offset by higher growth, like an anti-long-run-risk model.

- Log SDF:

$$m_{t+1} = -r_f - \frac{1}{2} x_t^2 - x_t \varepsilon_{d,t+1},$$

with price of risk process

$$x_{t+1} = (1 - \phi_x) \bar{x} + \phi_x x_t + \sigma_x \varepsilon_{x,t+1},$$

where $\varepsilon_{x,t+1}$ is uncorrelated with other shocks. Thus only dividend shocks are priced (since they enter the SDF), while shocks to the price of risk (risk aversion) are *unpriced*. This is hard to generate in GE (see Marfè, 2017, for one attempt): if risk aversion is time-varying, this will affect return, and usually this would affect prices.

The Lettau-Wachter Model

- ▶ Pricing of dividend strips: Price strip as “zero-coupon” equity paying dividend D_{t+n} , following recursion

$$P_{nt} = E_t[M_{t+1}P_{n-1,t+1}],$$

boundary condition $P_{0t} = D_t$ (discrete-time equivalent of PDE).

- ▶ Guess and verify exponentially affine solution

$$\frac{P_{nt}}{D_t} = \exp \{A(n) + B_x(n)x_t + B_z(n)z_t\}.$$

- ▶ Super straightforward to solve.

The Term Structure of Equity in the Lettau-Wachter Model

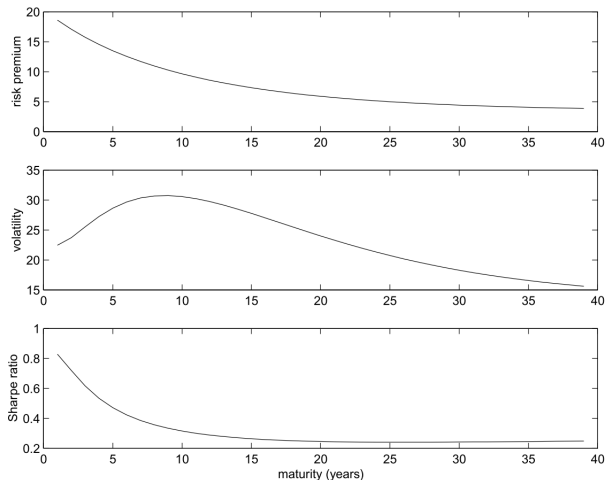


Figure 4. Summary statistics for zero-coupon equity. The top panel shows risk premia $E[R_{nt} - R^f]$ on zero-coupon equity over the risk-free rate. The middle panel shows the standard deviation of returns on zero-coupon equity. The bottom panel shows the Sharpe ratio (the risk premium divided by the standard deviation). Returns are simulated at a quarterly frequency and aggregated to an annual frequency.

CAPM Alphas in the Lettau-Wachter Model

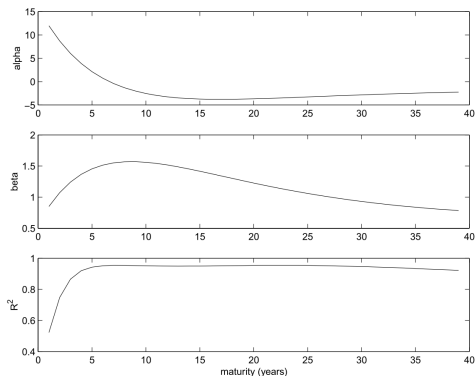


Figure 5. CAPM regressions for zero-coupon equity. The top panel shows the intercept from time-series regressions of excess zero-coupon equity returns on the excess market return, the middle panel shows the slope coefficient, and the bottom panel shows the R^2 . Statistics are shown as a function of maturity. Returns are simulated at a quarterly frequency and aggregated to an annual frequency.

- Clear implications for cross-sectional pricing patterns, which they pursue formally in the paper by specifying firms as claims on agg. dividends at different horizons.

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Thank you for a fantastic semester!!

... and please complete your course evaluations as soon as possible!

(It really does matter.)

<https://edu-apps.mit.edu/subjeval/studenthome.htm>