Model-Free International Stochastic Discount Factors

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Discussion:

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Background

Exchange-rate puzzles:

1. Forward premium puzzle
2. With complete markets, exchange rates are "too smooth" unless we think risk-sharing is nearly perfect [Brandt, Cochrane, Santa-Clara (2006)]
   - For any foreign (non-US) asset with return $R_{f,i}$,

     $$\mathbb{E}[M_f R_{f,i}] = 1$$
     $$\mathbb{E}[M_d X R_{f,i}] = 1,$$

     where $X \equiv S_{t+1}/S_t$ is exchange-rate change ($\uparrow$ means foreign appreciates relative to US), M_d is US-investor SDF
   - With unique SDFs, $X = M_f/M_d$, so $\text{Var}(x) = \text{Var}(m_f - m_d)$
   - We know $\text{Var}(m_f), \text{Var}(m_d)$ are very high (equity premium + forward premium $\iff$ high Sharpe ratios available), so they must covary strongly given exchange-rate volatility of 15%/year
3. Cyclicality puzzle [Backus, Smith (1993)]
   - Exchange rates don't comove empirically with proxies for relative macro conditions, even though (from above) $\text{Cov}(x, m_f - m_d)/\text{Var}(x) = 1$

Background

Standard approach to rationalize puzzles:

- Assume the existence of some "dark matter"
- For example, highly correlated long-run risks imply large Sharpe ratios, smooth exchange rates, and exchange rate comovement with hard-to-measure expectations of long-run consumption growth

This paper's approach:

- Step back from strict parameterizations of preferences and fundamentals
- Instead, consider what we learn by semiparametrically characterizing certain SDFs under different assumptions about market segmentation

What I'll do

Interesting and important set of questions

Discussion: Review step by step, with short comments/questions as I go

1. Theory
2. Empirical implementation
3. Results

Outline

1. Theory
2. Empirical Implementation
3. Results

Theoretical Setting

Work under null of:

1. Incomplete markets
   - Non-unique SDFs, so get an additional degree of freedom ("wedge") in matching exchange-rate returns [Backus, Foresi, Telmer (2001)]

$$x = m_f - m_d + \eta$$

   - Wedge isn't unrestricted (e.g., orthogonal to asset returns)

2. *Integrated* markets:

$$\text{Span}(\text{domestic returns}) = \text{Span}(\text{foreign returns} \times \text{exchange-rate change})$$

   - But I thought we wanted to know what happens when markets are segmented?
   - Response: By characterizing set of SDF processes under integrated markets, can hope to draw (contrapositive) conclusions about necessity of segmented markets if those processes are "unreasonable"

Theoretical Setting

Toolkit:

1. Solve for *minimum-dispersion* SDFs
   - This problem is a bit convoluted — $\min_M \log \mathbb{E}[M^\alpha]/(\alpha(\alpha-1))$. subject to pricing equation
   - But in practice, the authors consider just two such solutions: (i) minimum entropy ($\alpha = 0$), and (ii) minimum variance ($\alpha = 2$)
   - Proposition 1 gives "cookbook" for doing so given observed returns & α
   - Important: Minimum-variance SDF is the *unique* SDF in return space $\iff$ gives projection of "true" SDF onto return space
     - Why? $\mathbb{E}[MR] = 1 \iff \mathbb{E}[(M+\varepsilon)R] = 1$ if $\mathbb{E}[\varepsilon R] = 0$, so $\varepsilon = 0$ gives lowest-variance SDF and further has a unique solution [Cochrane (2005)]

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2. Consider some restrictions on exchange-rate wedge η for $\alpha = 0, \alpha = 2$
   - $\eta = 0$ for $\alpha = 0$: Even with incomplete markets, minimum-entropy SDF is inverse of growth-optimal portfolio, which can be expressed in either domestic or foreign currency, so we're stuck with $x = m_f - m_d$
   - $\eta = 0$ for $\alpha \neq 0$ iff $\text{Span}(\text{domestic returns}) = \text{Span}(\text{foreign returns})$; otherwise, there are unspanned exchange-rate risks
3. Decompose SDF into permanent/transitory components

Theory: Interpretation and Comments

Interpretation:

1. There are lots of (infinitely many) SDFs in incomplete markets. How to interpret the series of minimum-dispersion SDFs that the authors solve for?
   - Partial answer: These solutions by design give us conservative estimates of the moment being minimized
   - But what about the other moments? Are we over- or underestimating the correlation between domestic & foreign SDFs? The cyclicality of the wedge? . . .
2. "Unspanned" exchange-rate risks: Exchange rate fluctuates based on innovations to $M(\text{min. entropy}) - M(\text{min. variance})$ in both countries, which is orthogonal to traded returns
   - Direction and economic intuition a bit unclear
   - $X \uparrow$ when foreign unspanned risk is "worse," in order to compensate domestic investors for taking on that risk?

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Empirics

- Integrated but incomplete markets benchmark is that investors have access to aggregate equity, 10-year ($\approx \infty$-year) bonds, and short-term bonds in all (8) countries
  - US is domestic, average of all others is foreign
- This seems substantive and important: minimum-variance SDF depends on highest attainable Sharpe ratio, which of course depends on the set of assets you allow people to trade (and on the sample)
  - Sophisticated investors can access nonlinear foreign portfolios using derivatives
  - What's the covariance of state-price densities of domestic vs. foreign stocks? Would seem to give valuable information about shared risks
  - Instead, we're left to decide what a "reasonable" amount of risk-sharing vs. wedge volatility is when explaining exchange rate smoothness
- Segmented alternative: each country's investors trade in their own 3 assets, plus short-term bond in other countries

Sensitivity

Table 2. Properties of SDFs (Integrated Markets)

The table reports joint sample moments of the SDF and its components. Panel A reports statistics with respect to the minimum-entropy SDFs ($\alpha = 0$) and Panel B for minimum variance SDFs ($\alpha = 2$), $i = d, f$, $j = d, f$, $i \neq j$. The SDFs are derived when international trading is unrestricted, i.e. the financial markets are integrated. There is a US domestic SDF for each bilateral trade. We use monthly data from January 1975 to December 2015.

	US	UK	US	CH	US	JP	US	EU	US	AU	US	CA	US	NZ
	Panel A: $\alpha = 0$ (minimum entropy)													
$\mathbb{E}[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.841	0.872	0.979	0.926	0.740	0.694	0.690	0.681	0.919	0.951	0.726	0.720	0.639	0.557
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.917	0.948	1.048	0.951	0.814	0.707	0.774	0.725	1.029	1.065	0.823	0.827	0.681	0.625
$\sqrt{\text{Entropy}(M_i)}$	0.684	0.703	0.795	0.753	0.687	0.636	0.604	0.585	0.732	0.702	0.618	0.616	0.581	0.519
corr(M_i^T, M_i^P)	-0.454	-0.498	-0.407	-0.233	-0.519	-0.155	-0.549	-0.502	-0.411	-0.636	-0.506	-0.607	-0.317	-0.634
corr(M_i, M_j)		0.992		0.989		0.989		0.985		0.992		0.994		0.981
	Panel B: $\alpha = 2$ (minimum variance)													
$\mathbb{E}[M_i]$	0.982	0.973	0.982	0.990	0.982	0.991	0.982	0.980	0.982	0.966	0.982	0.973	0.982	0.956
Std(M_i)	0.739	0.754	0.873	0.834	0.699	0.658	0.639	0.622	0.776	0.791	0.659	0.655	0.600	0.535
Std(M_i^T)	0.120	0.122	0.120	0.061	0.120	0.091	0.120	0.068	0.120	0.107	0.120	0.111	0.120	0.091
Std(M_i^P)	0.803	0.824	0.930	0.853	0.763	0.670	0.711	0.659	0.839	0.874	0.733	0.735	0.632	0.595
corr(M_i^T, M_i^P)	-0.517	-0.587	-0.455	-0.268	-0.552	-0.169	-0.597	-0.564	-0.500	-0.775	-0.566	-0.683	-0.340	-0.665
corr(M_i, M_j)		0.989		0.988		0.989		0.984		0.988		0.993		0.979

- Disaggregated integrated-market results across countries
- Focus on second row in panel B: US columns show that estimated minimum SDF volatility ranges from 0.6 to 0.87 (increase of 45%) depending on the foreign country considered

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Results

Results on exchange-rate puzzles:

1. Forward premium puzzle
   - Estimated SDFs price carry returns by design, so this gets taken care of "for free"
2. Exchange rate smoothness
   - With integrated markets, even with XR wedge (minimum-variance case), need nearly perfect correlation between domestic & foreign SDFs (perfect risk-sharing) to explain the data. (Holds in general?)
   - Segmented markets: Lower SDF volatility (almost mechanically); less SDF comovement; higher wedge
   - Is this a win for the segmented-markets model?
3. Cyclicality puzzle
   - Still have $\text{Cov}(x, m_f - m_d)/\text{Var}(x) \approx 1$
   - So the response to the puzzle here is either: (i) consumption is the wrong proxy for the SDF; (ii) it could be the right proxy at short horizons (captured as temporary component as SDF, which exhibits acyclicality w.r.t. x), but the permanent component is what matters

Final Notes

- Really interesting paper, getting at important questions
- Lots of other stuff (including on possible importance of intermediaries) I didn't even have time to touch on!
- Would love more on interpretation of minimum-dispersion SDFs — should we be taking them literally?
- Some room for additional empirical tests with more assets