Overinference from Weak Signals and Underinference from Strong Signals

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### Background

How does new information change beliefs? Conflicting evidence in different environments...

- 1. Financial markets
  - Excess volatility in aggregate valuations: Consistent with overreaction
  - Post-earnings drift for individual stocks: Consistent with underreaction
  - ▶ ...
- 2. The lab

  - Benjamin (2019) handbook chapter:

"Stylized Fact 1: Underinference is by far the dominant direction of bias."

Seek to reconcile evidence from both controlled experiments and observational data.

## This Paper

What drives over- vs. underinference?

- Depends on the strength of information
- Experiments overwhelmingly study strong signals
  - Benjamin (2019) meta-analysis: 500 experimental treatment blocks on inference from symmetric binary signal about binary state
  - ▶ None of these experiments features signal with  $\mathbb{P}(\text{high signal} | \text{high state}) < 0.6$
- Outside the lab, constant stream of weakly informative signals about future events
  - New poll about politician's favorability...reelection
  - Daily index return...annual index return
  - But can find counterexamples: earnings are informative about firm's fundamental value [Vuolteenaho 2002]

#### Underinference from strong signals, overinference from weak signals

#### What We Do

Systematic study of reaction to information of different strengths:

- 1. In the lab
  - Straightforward to vary signal strength given control of DGP
  - But concerns about external validity
- 2. In high-stakes observational data from betting & financial markets
  - ▶ No longer control DGP, so need ex ante empirical correlate of signal strength...
  - ...and well-defined measures of over- vs. underinference
- Simple theory to tie it together: Imprecision about signal strength
  - People have meta-prior on how much to update, don't fully adjust to true signal strength
  - ▶ Provides useful language for interpreting results, and ties together empirical evidence

## Contribution

#### Underinference:

- Dominant direction in balls-and-urns experiments (Benjamin 2019; Edwards 1968; Griffin & Tversky 1992; Enke & Graeber 2023)
  - But no prior work uses weak signals
  - One recent paper does, confirms patterns we find (Ba, Bohren, & Imas WP)
- Also: Neglect signal quantity or setting (Griffin & Tversky 1992; Massey & Wu 2005)

#### **Overinference:**

- More common in observational data (Bordalo et al. 2022; Bordalo et al. 2019)
  - Our argument: Environments with weakly informative signals
- Also common in forecasting experiments, varying horizon (Afrouzi et al. 2022; Fan et al. WP)
- Finance literature on overreaction and excess volatility (e.g. De Bondt and Thaler 1985)

We unite these strands: underinfer from strong signals and overinfer from weak signals:

- Strong: More common in lab. Weak: More common in forecast revisions.
- Other complementary mechanisms: Fan et al. WP; Kwon and Tang 2023

Combine experiment with betting + finance data for external validity (Levitt & List 2006)

- ▶ Use tools from Augenblick & Rabin (2021), Augenblick & Lazarus (WP)
- ▶ ID settings in which signals strong vs. weak by using time to resolution

#### Outline

1. Background

2. Theory

- 3. Experimental Evidence
- 4. Observational Data
- 5. Conclusions

## Theory: Setup

- Person has prior  $\pi_0 \equiv \mathbb{P}(\theta = 1)$  about binary state  $\theta \in \{0, 1\}$
- Sees binary signal  $s \in \{s_L, s_H\}$ , with  $\mathbb{P}(\theta = 1|s_H) > \mathbb{P}(\theta = 1|s_L)$
- Consider misperception of signal strength in person's subjective posterior  $\hat{\pi}_1$ :

$$\operatorname{logit}(\hat{\pi}_1) = \operatorname{logit}(\pi_0) + \frac{w}{w} \log \left( \frac{\mathbb{P}(s|\theta = 1)}{\mathbb{P}(s|\theta = 0)} \right)$$

- w = 1: Bayesian ( $\hat{\pi}_1 = \pi_1$ )
- w > 1: Overinference
- ► *w* < 1: Underinference

**Main question:** How does *w* depend on signal strength  $\mathbb{S} \equiv \left| \log \left( \frac{\mathbb{P}(s|\theta=1)}{\mathbb{P}(s|\theta=0)} \right) \right|$ ?

# Misweighting via Imprecision

#### **Cognitive imprecision:**

- Longstanding idea (Weber 1834; Fechner 1860), recent applications to numerical values, probabilities, . . . (Woodford 2020; Khaw et al. 2021; Frydman & Jin 2022; Enke & Graeber 2023)
- We extend to imprecision about strength of information S
- ▶ Basic idea: Noisy representation of signal strength → shrinkage to "moderate" strength
  - ▶ Weak signals appear "too weak" relative to prior signal distribution → update too much
  - Consistent with inattention models (Gabaix 2019)

# Misweighting via Imprecision

#### **Cognitive imprecision:**

- 1. Cognitive prior about signal strengths:  $\log S \sim \mathcal{N}(\mu_0, \sigma^2)$
- + 2. Signal of strength  $\mathbb{S} \longrightarrow$  noisy cognitive representation *r*, with  $r \sim \mathcal{N}(\log \mathbb{S}, \eta^2)$ 
  - Easier to differentiate S and  $S + \epsilon$  when S small
  - $\Rightarrow$  Posterior mean of perceived S follows power law  $\mathbf{k} \cdot \mathbb{S}^{\beta}$ , with  $\beta \equiv \frac{\sigma^2}{\sigma^2 + n^2} \in (0, 1)$ 
    - Shrinkage to "moderate" signal strength
    - Switching point S\* ≡ k<sup>1/(1-β)</sup> s.t. person underestimates S iff S > S\*: w > 1 if S < S\* (overweight weak signals + overinference) w < 1 if S > S\* (underweight strong signals + underinference)

- y-axis: weight w(p) put on signal of precision p relative to Bayesian
- Compare: **Bayes** (w = 1)



- y-axis: weight w(p) put on signal of precision p relative to Bayesian
- Compare: **Bayes** (w = 1), constant underweight



- y-axis: weight w(p) put on signal of precision p relative to Bayesian
- ▶ Compare: **Bayes** (*w* = 1), constant underweight, constant overweight



- y-axis: weight w(p) put on signal of precision p relative to Bayesian
- ▶ Compare: Bayes (*w* = 1), constant underweight, constant overweight, our model



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### Preregistered Experiment

- ▶ n = 552 U.S. adults recruited from Prolific (prolific.co) in March 2021
- Standard bookbag-and-poker-chips setup: Card drawn from either green or purple deck (equally likely ex ante)
- Binary signal about deck: Card suit
- ▶ Green deck has share  $p_1$  diamond  $\diamondsuit$  cards &  $1 p_1$  spades  $\blacklozenge$  (Purple:  $p_2, 1 p_2$ )
- ▶ Baseline: Symmetric signals, precision  $p \equiv p_1 = 1 p_2$
- Elicit  $\mathbb{P}(\text{deck} \mid \text{card drawn})$ , compare answers as we vary p (via deck composition)
- Answers incentivized (based on probability points), keep only if pass attention check (91%)

## Example Page

You draw a card from one of two modified decks of cards; a **Green** deck or a **Purple** deck.

The Green deck has 853 Diamonds ( $\blacklozenge$ ) and 812 Spades ( $\spadesuit$ ). The Purple deck has 812 Diamonds ( $\diamondsuit$ ) and 853 Spades ( $\spadesuit$ ).

The card you draw is a **Spade** ( $\blacklozenge$ ).

Please answer between 0 and 100 for each question and have your answers sum up to 100, where higher numbers mean you think that deck is more likely. Your answer may include decimals.

Percent chance that your $\mathbf{Spade}$ ( $\bigstar$ ) came from the $\mathbf{Green}$ deck	0	percent
Percent chance that your ${f Spade}(\spadesuit)$ came from the ${f Purple}$ deck	0	percent
Total	0	percent

Vary deck size (small/large), suit (diamond/spade), signal strength (strong/weak)

- y-axis: weight w(p) put on signal of precision p
- Compare model (black curve) to Bayes (w = 1), constant overweight, constant underweight



## Over- and Underinference by Signal Strength: Main Results



Replicate previous experimental results with strong signals...

## Over- and Underinference by Signal Strength: Main Results



Replicate previous experimental results with strong signals...

...but strong evidence for underinference with weak signals

#### Model Estimate

Recall: model predicts people perceive signal strength  $\mathbb{S} \equiv \left| \log \left( \frac{P(s|\theta=1)}{P(s|\theta=0)} \right) \right|$  as strength  $\hat{\mathbb{S}} = k \cdot \mathbb{S}^{\beta}$ 

Nonlinear least squares estimate: k = 0.88 (s.e. 0.02);  $\beta = 0.76$  (s.e. 0.03)

Switching point  $\mathbb{S}^*$  is such that  $\hat{\mathbb{S}}^* = \mathbb{S}^*$ , occurs when signal precision  $P(s_H | \theta = 1) = 0.64$ :

- Signals with precision  $> 0.64 \rightarrow \hat{\mathbb{S}} < \mathbb{S}$ , i.e. underinference
- Signals with precision  $< 0.64 \rightarrow \hat{\mathbb{S}} > \mathbb{S}$ , i.e. overinference

Explains why hard to detect overinference when looking at signals with precision  $\geq 0.6$ 

# Heterogeneity and Learning



Lower score on cognitive reflection test (Frederick 2005): Infer more from weak signals and less from strong signals

# Heterogeneity and Learning



Less task experience: Infer more from weak signals and less from strong signals

# Additional Analysis and Robustness

#### In paper (and appendix):

- Additional heterogeneity analyses to speak to psychological channel: Heterogeneity by variance in signal weights, news consumption
- Excessive willingness to pay for weak signals, too little for strong signals
- Asymmetric signals
- Multiple signals
- Tests of potential alternative explanations
- Formal regression evidence on main results

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## Evidence from Options & Betting Markets

- Experiment is tightly controlled, but unclear external validity
- Turn to high-stakes real-world setting: market-implied probabilities in betting markets and index options
- Challenges:
  - 1. No longer control DGP ⇒ true signal informativeness is unknown, as is objectively correct belief period by period
  - 2. Interpretation as "beliefs"?
- Develop & argue for proxy for signal informativeness based on time to resolution
  - ► Key feature of setting: Fix expiration date and observe implied beliefs over time within contract

### Belief Movement and Uncertainty Reduction

For belief  $\pi_t$  over binary state  $\theta$  ("Bulls win") realized at *T*:

- **Belief movement** (amount of updating):  $m_{t,t+1} = (\Delta \text{ belief})^2$
- Uncertainty reduction (signal informativeness):  $r_{t,t+1}$  = decrease in subjective variance from t to t + 1
- **Result** (Augenblick & Rabin 2021): For a Bayesian, for any DGP,  $\mathbb{E}_t[m_{t,t+1}] = \mathbb{E}_t[r_{t,t+1}]$ .
  - Crucial features: Valid regardless of DGP, for any subset of data (as long as cut is ex ante)

#### **Our hypothesis:**

- Signals are weak  $\implies \mathbb{E}[m] > \mathbb{E}[r]$  (overinference); signals strong  $\implies \mathbb{E}[m] < \mathbb{E}[r]$  (underinf.)
- Sorting variable for signal strength: time to resolution
  - Ex ante known and observable
  - And signals are intuitively much more informative about payoffs closer to expiration
- To verify that (1) overinference  $\iff \mathbb{E}[m] > \mathbb{E}[r]$ , and (2) stronger signals closer to resolution: theory & **simulations**

## Simulated Belief Streams



- Simulate one million "games" with two teams
  - One team scores in each of 27 periods (50/50), winner has most points at end
  - Movement (sq. chg. in beliefs) & uncertainty reduction (drop in variance)
- Our model

(calibrated from experiment): m > r early and m < r late

#### Data

- 1. Sports betting (Moskowitz 2021): Betfair
  - Large UK-based prediction exchange matching individual bettors on contracts over game outcome
  - Within-game binary bet prices for soccer, American football, baseball, basketball, hockey games
- 2. Index options:
  - OptionMetrics: Daily S&P index option prices traded on CBOE, 1996–2018 (Augenblick & Lazarus)
  - ▶ Use risk-neutral beliefs over 5-percentage-point return ranges, keeping at-the-money +/- 20 ppt
  - Also translate RN beliefs to **physical** beliefs under hundreds of parameterizations for risk aversion  $\phi_{t,j}$  (nearly identical results)

#### Betfair Data: NBA and NFL

Using implied probabilities from Betfair over the course of sports games:



Over course of game, *m* and *r* increase, and m - r moves from + to -.

## Betfair Data: All Sports



Over course of game, *m* and *r* increase, and m - r moves from + to -.

#### Data

- 1. Sports betting (Moskowitz 2021): Betfair
  - Large UK-based prediction exchange matching individual bettors on contracts over game outcome
  - ▶ Within-game binary bet prices for soccer, American football, baseball, basketball, hockey games
- 2. Risk-neutral beliefs from index options:
  - OptionMetrics: Daily S&P index option prices traded on CBOE, 1996–2018 (Augenblick & Lazarus)
  - ▶ Use risk-neutral beliefs over 5-percentage-point return ranges, keeping at-the-money +/- 20 ppt
  - Augenblick & Lazarus (2022) show how risk-neutral beliefs are informative about excess vol... less important for our analysis, because looking for relative over-/underreaction within a contract
  - Also translate RN beliefs to **physical** beliefs under hundreds of parameterizations for risk aversion  $\phi_{t,j}$  (nearly identical results)

## **Options** Data



As move closer to expiration, *m* and *r* increase, and m - r moves from + to -. Regression evidence that *m* increases less than one-for-one w/ informativeness: Appendix

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#### Conclusions

- Updating behavior differs by signal strength: overinference from weak signals & underinference from strong signals
- Robust & consistent evidence across domains: experiment, sports betting, financial markets
- ▶ In line with predictions of simple theoretical framework
- ▶ Reconciles disparate findings in experimental lit. & finance lit.
- Interesting open questions on how people form cognitive defaults in novel settings & what structure the noise in signal processing takes

# APPENDIX

## Heterogeneity and Learning



▶ Higher variance in signal weights: Infer more from weak signals, less from strong signals

- Suggests cognitive imprecision may be driving biases
- Though part of effect may be mechanical

## Effects on Overinference: Linear Regression Specification

	(1)	(2)	(3)	(4)	(5)
	Main Effects	By CRT Score	By Round	By Noise	By News
Signal Strength	-0.308	-0.481	-0.578	-0.145	-0.519
	(0.031)	(0.064)	(0.072)	(0.059)	(0.093)
Strength x CRT Score		0.102			
		(0.028)			
Strength x Round Number			0.042		
-			(0.009)		
Strength x Noise				-0.349	
				(0.138)	
Strength x News Cons					0.334
-					(0.129)
Constant	1.420	1.421	1.416	1.404	1.420
	(0.030)	(0.030)	(0.030)	(0.042)	(0.030)
Subject FE	Yes	Yes	Yes	Yes	Yes
Round FE	Yes	Yes	Yes	Yes	Yes
Observations	3964	3964	3964	1901	3964
R <sup>2</sup>	0.23	0.23	0.24	0.29	0.23

Standard errors in parentheses

### Effects on Overinference: Power Law Regression Specification

DV: Weight on signal	(1)	(2)	(3)	(4)	(5)
	Main Effects	By CRT Score	By Round	By SD	By News
Switching point <i>p</i> *	0.644	0.624	0.628	0.702	0.628
	(0.011)	(0.013)	(0.013)	(0.014)	(0.015)
Sensitivity $\beta$	0.761	0.741	0.745	0.803	0.745
	(0.028)	(0.034)	(0.035)	(0.028)	(0.036)
$\beta_1 \ge CRT$ Score		0.024			
		(0.007)			
$\beta_1$ x Round Number			0.005		
			(0.002)		
$\beta_1 \ge SD$ of Guesses				-0.198	
				(0.044)	
$\beta_1$ x News Cons					0.054
					(0.027)
Observations	3964	3964	3964	3964	3964
$R^2$	0.38	0.38	0.38	0.38	0.38

Standard errors in parentheses

The function estimated is  $\frac{logit(p^*)^{1-\beta}}{logit(\pi_1)^{1-\beta}}$  for (1) and  $\frac{logit(p^*)^{1-\beta-\beta_1\text{-Interaction}}}{logit(\pi_1)^{1-\beta-\beta_1\text{-Interaction}}}$  for each interaction from (2)-(5).

#### Demand for Information

- Mistaken demand for information often tracks inference biases (Ambuehl and Li 2018)
- ▶ If demand tracks misinference, people will:
  - Pay too much for weak signals
  - Pay too little for strong signals
  - Generally: have demand that is too insensitive to the quality of the information

#### Demand for Information



WTP bias similar to beliefs:

- Excess demand for weak signals
- Too little demand for strong signals

Effects even more severe for WTP:

- Explanation: Subjects also undervalue multiple signals.
  - Run main treatment with 3 signals and see this effect. (Details)

#### **Alternative Explanations**

Misperceptions of probabilities rather than signal strengths (relative probabilities)?

- E.g. Maybe people overestimate probabilities that are  $1/2 + \epsilon$  vs.  $1/2 \epsilon$ .
- ▶ Test using **asymmetric** signals, where one signal is close to uninformative
- $P(s|\theta = 1) \gg 1/2$ , but  $P(s|\theta = 0) = 1/2 + / -\epsilon$ .
- Whether + or  $\epsilon$  would affect agent who misperceives probabilities, but not agents in our model
- Find little effect of changing + to  $\epsilon$ . Details
- ▶ Others: Aversion to 50%, # of cards, preference for one deck or color
  - No evidence these are driving results
  - ▶ E.g. 72 subjects see decks with P(Green) = 1/2 and 69 (96%) have posteriors of exactly 1/2

## Underinference from Multiple Signals



Effect of signal strength similar across groups, but switching point changes:

- **Gray:** 0.644
- ▶ Purple: 0.523
- Green: 0.507

Consistent with Griffin and Tversky (1992)

## Asymmetric Signals

Signal in one state has likelihood far from 1/2:

• Vary  $P(s|\theta = 1) = 0.65$  or 0.80

Signal in other state has likelihood close to 1/2:

Vary  $P(s|\theta = 0) = 0.495$  or 0.505

- Estimate  $\beta$  from power law model for each  $P(s|\theta = 0)$
- If effects due to probabilities, would expect significantly higher  $\beta$  when  $P(s|\theta = 0) = 0.505$
- ► Instead:  $\beta = 0.54$  when  $P(s|\theta = 0) = 0.505$  and  $\beta = 0.60$  when  $P(s|\theta = 0) = 0.495$ 
  - ▶ Difference small, in the opposite direction, and not statistically significant (p-value = 0.404)









## Formal Regression Framework

- Our hypothesis:
  - Signals are weak  $\implies \mathbb{E}[m] > \mathbb{E}[r]$
  - Signals are strong  $\implies \mathbb{E}[m] < \mathbb{E}[r]$
- Sorting variable for signal strength: time to resolution
- Cut market-implied belief streams into 24 time windows
  - Paper has results for alternative choices
- Regress average movement on average uncertainty reduction within each window
  - If used individual observations, would have severe attenuation bias
  - ▶ Bayesian:  $\mathbb{E}_t[m_{t,t+1}] = \mathbb{E}_t[r_{t,t+1}] \implies$  should observe intercept of 0, slope of 1
  - Our theory: positive intercept, slope below 1
  - Results strongly align with our theoretical predictions

# Formal Regression Evidence

#### **Regressions of Movement on Uncertainty Reduction**

Dep Var:	Betting				Options		
Movement	Soccer	Basketball	Baseball	Hockey	Football	Raw	Risk-Adj.
Uncert. Red.	0.918	0.806	0.889	0.945	0.912	0.680	0.733
	(0.005)	(0.008)	(0.013)	(0.013)	(0.027)	(0.040)	(0.041)
Constant	0.0009	0.0018	0.0026	0.0018	0.0015	0.0065	0.0060
	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0002)	(0.0003)	(0.0003)
<i>R</i> <sup>2</sup>	0.977	0.985	0.995	0.976	0.995	0.944	0.941
Time Chunks	24	24	24	24	24	24	24
Events	6,584	5,176	3,927	4,123	1,390	955	955
Observations	4,589,289	867,567	166,346	109,751	86,193	58,864	58,864
<i>p</i> -val.: $\beta_1 = 1$	< 0.001	< 0.001	< 0.001	< 0.001	0.007	< 0.001	< 0.001
<i>p</i> -val.: $\beta_0 = 0$	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001

Parentheses show bootstrapped standard errors with resampling clustered by stream.

Overreaction when signal informativeness is low...but decreases systematically as signal strength increases, with underreaction close to expiration. 12 time chunks 36 time chunks

#### Robustness: 12 Time Periods

#### **Regressions of Movement on Uncertainty Reduction**

Dep Var:	Sports				Finance		
Movement	Soccer	Basketball	Baseball	Hockey	Football	Raw	Risk-Adj.
Uncert. Red.	0.839	0.797	0.903	0.987	0.912	0.796	0.861
	(0.006)	(0.007)	(0.012)	(0.012)	(0.027)	(0.054)	(0.063)
Constant	0.0014	0.0018	0.0024	0.0013	0.0015	0.0060	0.0054
	(0.0003)	(0.0003)	(0.0004)	(0.0009)	(0.0002)	(0.0005)	(0.0005)
<i>R</i> <sup>2</sup>	0.984	0.991	0.996	0.990	0.997	0.945	0.941
Time Chunks	12	12	12	12	12	12	12
Events	6,584	5,176	3 <i>,</i> 927	4,123	1,390	955	955
Observations	4,589,289	867,567	166,346	109,751	86,193	58,864	58,864
<i>p</i> -val.: $\beta_1 = 1$	< 0.001	< 0.001	< 0.001	0.274	0.002	0.004	0.025
<i>p</i> -val.: $\beta_0 = 0$	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001	< 0.001

Parentheses show bootstrapped standard errors with resampling clustered by stream.

 Overreaction when signal informativeness is low...but decreases systematically as signal strength increases, with underreaction close to expiration

#### Robustness: 36 Time Periods

#### **Regressions of Movement on Uncertainty Reduction**

Dep Var:	Sports				Finance		
Movement	Soccer	Basketball	Baseball	Hockey	Football	Raw	Risk-Adj.
Uncert. Red.	0.847	0.849	0.883	0.925	0.920	0.705	0.751
	(0.003)	(0.008)	(0.015)	(0.013)	(0.026)	(0.035)	(0.040)
Constant	0.0014	0.0016	0.0027	0.0020	0.0015	0.0063	0.0058
	(0.0001)	(0.0001)	(0.0002)	(0.0002)	(0.0001)	(0.0003)	(0.0003)
<i>R</i> <sup>2</sup>	0.955	0.974	0.993	0.975	0.982	0.932	0.928
Time Chunks	36	36	36	36	36	36	36
Events	6,584	5,176	3,927	4,123	1,390	955	955
Observations	4,589,289	867,567	166,346	109,751	86,193	58,864	58,864
<i>p</i> -val.: $\beta_1 = 1$	< 0.001	< 0.001	< 0.001	< 0.001	0.054	< 0.001	< 0.001
<i>p</i> -val.: $\beta_0 = 0$	< 0.001	< 0.001	< 0.001	< 0.001	0.051	< 0.001	< 0.001

Parentheses show bootstrapped standard errors with resampling clustered by stream.

 Overreaction when signal informativeness is low...but decreases systematically as signal strength increases, with underreaction close to expiration