Forward Return Expectations

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Background

Well-studied set of questions:

- What is the expected excess return on the market?
- How does it evolve over time?
- Are there systematic errors in return predictions?

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$$p_t - d_t = \kappa - \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1}}_{} + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}$$

$$\mathbb{E}_t r_{t+1} + \underbrace{\textstyle\sum_{j=1}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1}}_{}$$

much more important for pricing!

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$$p_t - d_t = \kappa - \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1}}_{\mathbb{E}_t r_{t+1} + \sum_{j=1}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1}} + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}$$

Our focus:

- What is the **expected future equity premium**?
- How does it compare to the *actual* future equity premium $\mathbb{E}_{t+j} r_{t+j+1}$?
- Are there systematic errors in *expected* return predictions?

What We Do

1. Measure equity premium at multiple horizons n (using options or surveys):

$$\textbf{Spot rate:} \quad \mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n} - r^f_{t,t+n}]$$

2. Calculate expected future equity premium:

$$\textbf{Forward rate:} \quad f_t^{(n)} = \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(1)}]$$

3. Compare forward rate to realized future spot rate:

$$\textbf{Forecast error:} \quad \varepsilon_{t+n} = \mu_{t+n}^{(1)} - f_t^{(n)}$$

What We Do

$$
\begin{aligned}
\textbf{Spot rate:} \quad \mu_t^{(n)} &= \mathbb{E}_t[r_{t,t+n} - r^f_{t,t+n}] \\
\textbf{Forward rate:} \quad f_t^{(n)} &= \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t[\mu^{(1)}_{t+n}] \\
\textbf{Forecast error:} \quad \varepsilon_{t+n} &= \mu^{(1)}_{t+n} - f_t^{(n)}
\end{aligned}
$$

Measurement:

1. Option prices
   - Measurement of log equity premium
   - Forecast errors identified under much weaker conditions than expected returns themselves
   - We can test whether expectations are intertemporally consistent, without needing to take a stand on whether spot expected returns are themselves rational
   - Rich data...but ultimately model-based
2. Survey expectations
   - Term structure of expected returns in Livingston and Duke-CFO survey
   - Model-free tests...but not as rich data

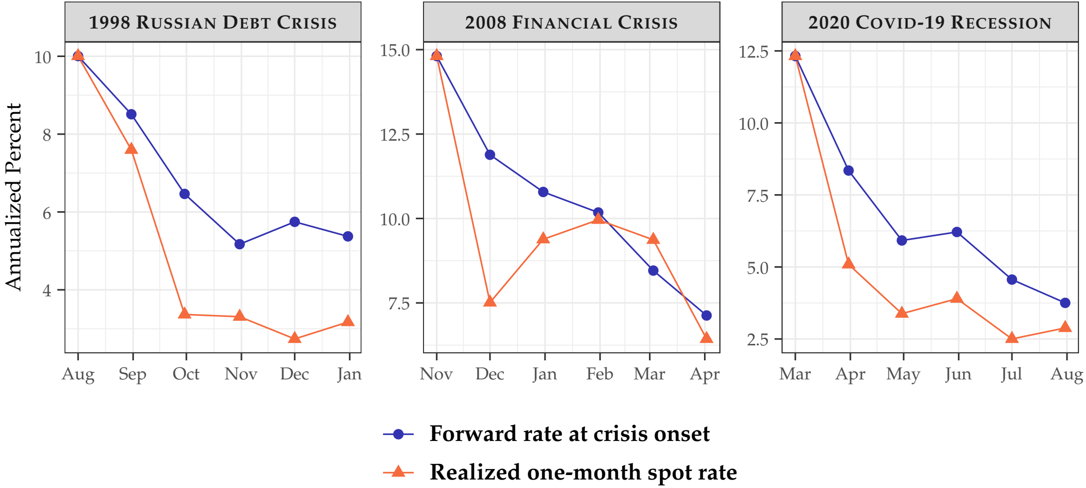
What We Find

$$
\begin{aligned}
\textbf{Spot rate:} \quad \mu_t^{(n)} &= \mathbb{E}_t[r_{t,t+n} - r^f_{t,t+n}] \\
\textbf{Forward rate:} \quad f_t^{(n)} &= \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(1)}] \\
\textbf{Forecast error:} \quad \varepsilon_{t+n} &= \mu_{t+n}^{(1)} - f_t^{(n)}
\end{aligned}
$$

Excess countercyclicality in forward return expectations:

1. In options & surveys, forward rates are countercyclical...
   - When the market $\searrow \Longrightarrow$ expectations of future equity premia $\nearrow$
   - Contrasts with short-horizon extrapolation in some surveys [Greenwood & Shleifer 2014]
2. ...and in fact **too countercyclical**
   - In bad times, investors believe expected returns will stay elevated for longer and by more than their own subsequent beliefs justify (vice versa in good times)
   - Thus excessively cyclical (and excessively volatile) forward return expectations

Illustration: Option-Based Forward and Realized Spot Rates in Crises



Summary of Evidence

	Expectations Measured by:		
	Options	Livingston Survey	CFO Survey
Panel A. Predictability in Spot Rates ($\mu_{i,t+1} = \beta_0 + \beta_1 f_{i,t} + e_{i,t+1}$)			
β_1	0.88	0.68	0.63
R^2	0.71	0.38	0.46
Panel B. Predictability of Forecast Errors ($\epsilon_{i,t+1} = \beta_0 + \beta_1 f_{i,t} + e_{i,t+1}$)			
β_1	-0.34	-0.19	-0.15
R^2	0.06	0.06	0.03
Panel C. Cyclical Variation in Forward Rates and Forecast Errors			
$\rho\,(f_{i,t}, 1/\text{CAPE}_t)$	0.04	0.42	0.21
$\rho\,(\epsilon_{i,t+1}, 1/\text{CAPE}_t)$	-0.38	-0.19	-0.38

Implications

Excess cyclicality in forward return expectations helps us understand:

1. Excess volatility in stock prices
   - When prices are depressed, this partly reflects investors expecting persistently high risk premia
   - If investors didn't overestimate persistence, would see more modest fluctuations in prices (about 50% less during 2008 crisis, nearly 100% less during Covid crash)
2. Inelastic demand for equities [Gabaix & Koijen 2022]
   - Puzzle: Why investors change weight in equities so modestly in response to change in stock prices
   - Partial resolution: If price drop leads to increases in expected returns **mainly at long horizons**, shouldn't see big increase in portfolio weight
3. Facts about equity term structure
4. Debate on cyclicality of subjective risk premia

Roadmap

1. Introduction
2. Price-Based Measurement of Expectations: Theory
3. Evidence from Price-Based Expectations
4. Evidence from Survey-Based Expectations
5. Explaining Forecast Errors
6. Implications and Conclusions

Setting and Identification Challenge

- Representative agent ("the market")...or any unconstrained trader fully invested in the market
- **Building block**: **LVIX** $\mathcal{L}_t^{(n)}$ (Gao & Martin 2021):

$$\underbrace{\mathbb{E}_t[r_{t,t+n} - r^f_{t,t+n}]}_{\mu_t^{(n)}} = \underbrace{\mathbb{E}_t[M_{t,t+n}R_{t,t+n}r_{t,t+n} - r^f_{t,t+n}]}_{\mathcal{L}_t^{(n)}} - \underbrace{\mathrm{Cov}_t(M_{t,t+n}R_{t,t+n}, r_{t,t+n})}_{\mathcal{C}_t^{(n)}}$$

- $\mathcal{L}_t^{(n)}$: Observable from options
- $\mathcal{C}_t^{(n)} = 0$ under log utility ($MR = 1$)...otherwise introduces unobservable contamination
- Gao & Martin argue $\mathcal{C}_t^{(n)} \leqslant 0$...but what about for fwd rate $f_t^{(n,m)} = \mathcal{L}_t^{(n+m)} - \mathcal{L}_t^{(n)} + \mathcal{C}_t^{(n)} - \mathcal{C}_t^{(n+m)}$?
- **Key insight:** Covariance terms largely cancel when measuring **forecast errors** $\varepsilon_{t+n}^{(m)} = \mu_{t+n}^{(m)} - f_t^{(n,m)}$
- Option-based expected returns may not be good predictors of realized returns...

  ...but they should predict **themselves**

The Log-Normal Case: Result

Observable forecast-error proxy:

$$\widehat{\varepsilon}_{t+n}^{(m)} = \mathscr{L}_{t+n}^{(m)} - \mathscr{L}_t^{(n+m)} + \mathscr{L}_t^{(n)}$$

Result 1 (*Log-Normal Identification*)

For a general SDF $M_{t,t+n}$, assuming $M_{t,t+n}$, $R_{t,t+n}$ are jointly log-normal:

$$\mathbb{E}_t\left[\widehat{\varepsilon}_{t+n}^{(m)}\right] = \mathbb{E}_t\left[\varepsilon_{t+n}^{(m)}\right] - \text{Cov}_t(M_{t,t+n}R_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}])$$

- Covariance term now relates to pricing of *discount-rate risk*, rather than *realized-return* risk
- Likely much smaller than previous term: expected returns are much less volatile than realized returns
- Can be disciplined empirically or theoretically
- Basic idea of proof: $MR_{t,t+n}$ is orthogonal to unexpected component of $r_{t+n,t+n+m}$

The General Case: Result

Define forecast-error proxy and expected-return proxy:

$$\widehat{\varepsilon}_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} - \mathcal{L}_{t}^{(n+m)} + \mathcal{L}_{t}^{(n)}$$

$$\widehat{\mu}_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} + r_{t+n,t+n+m}^{f}$$

Result 2 (*Generalized Identification*)

For any SDF $M_{t,t+n}$,

$$\mathbb{E}_t\left[\widehat{\varepsilon}_{t+n}^{(m)}\right] = \mathbb{E}_t\left[\varepsilon_{t+n}^{(m)}\right] - \mathrm{Cov}_t\left(M_{t,t+n}R_{t,t+n}, \widehat{\mu}_{t+n}^{(m)}\right)$$

- Intuition from log-normal case carries over, with $\mathbb{E}_{t+n}[r_{t+n,t+n+m}]$ replaced by $\widehat{\mu}_{t+n}^{(m)}$
- LVIX-based $\widehat{\mu}_{t+n}^{(m)}$ is closely related to $\mathbb{E}_{t+n}[r_{t+n,t+n+m}]$ ...but $\widehat{\mu}_{t+n}^{(m)}$ is directly observable
- Main specification: $\widehat{\mu}_{t+n}^{(m)}$ is $\frac{1}{10}$ as volatile as realized return $r_{t+n,t+n+m}$

  $\Longrightarrow$ unobserved covariance likely much smaller for forecast errors than for spot rates

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Data and Measurement: Options

Data:

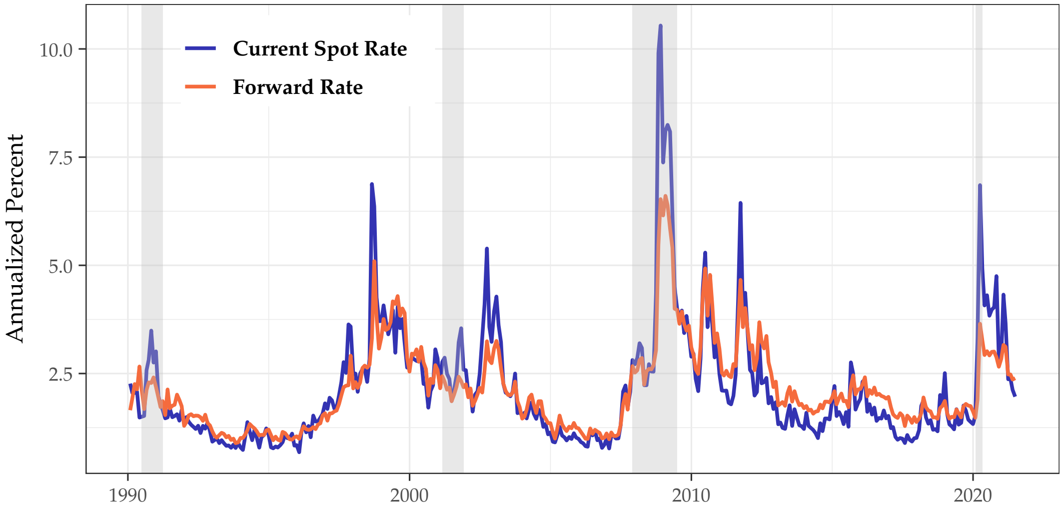
- Main data: Global panel of index options from OptionMetrics (monthly data, standard filters)
  - For U.S. sample: 1990–2021
  - For international sample: Consider 10 major indices, with data since at least 2006
- Sample monthly and apply standard filters
- Baseline: 6-month horizon, 6 months forward ($n = m = 6$)

Measuring LVIX: Following Gao & Martin (2021), Carr & Madan (2001),

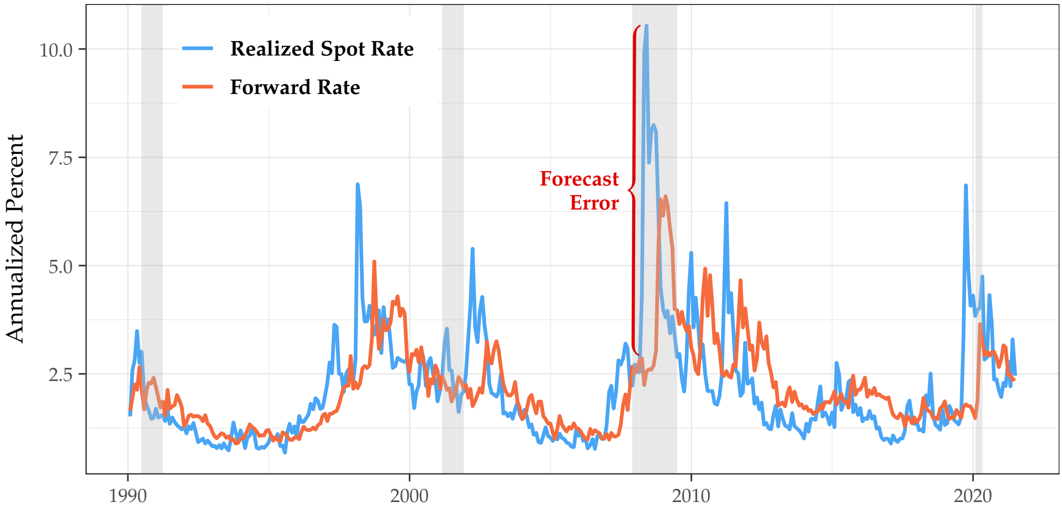
$$\mathcal{L}_t^{(n)} = (R^f_{t,t+n})^{-1}\,\mathbb{E}_t^*[R_{t,t+n} r_{t,t+n}] - r^f_{t,t+n} = \frac{1}{P_t}\left\{\int_0^{F_t^{(n)}} \frac{\text{put}_t^{(n)}(K)}{K} dK + \int_{F_t^{(n)}}^{\infty} \frac{\text{call}_t^{(n)}(K)}{K} dK\right\}$$

- Calculate integral a bunch of different ways
- First: Simplify by working under log assumption, so LVIX $\Longrightarrow$ spot & forward rates

Estimates: Contemporaneous U.S. Spot and Forward Rates



Estimates: Realized U.S. Spot and Forward Rates



Do Forward Rates Predict Future Spot Rates?

Mincer–Zarnowitz Regressions for Spot Rates by Country

$$\mu_{t+6}^{(6)} = \beta_0 + \beta_1 f_t^{(6,6)} + \epsilon_{t+6}$$

	(1) U.S.	**(2)** Ex-U.S.	**(3)** All
$f_t^{(6,6)}$	0.67***	0.55***	0.56***
	(0.096)	(0.056)	(0.055)
Intercept	0.74***		
	(0.28)		
Country FEs	✗	✓	✓
p-value: $\beta_1 = 1$	0.003	0.000	0.000
Obs.	378	1,849	2,227
R^2	0.22	0.21	0.22
Within R^2	—	0.14	0.15

SEs in (1) are heteroskedasticity and autocorrelation-robust [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Substantial predictive power…
- …but $\beta_1 \neq 1$, suggesting forward rates overshoot future spot rates
- What if β_1 estimate is downwardly biased due to measurement error?
- To address this, now consider IV using shorter-term forward rate $f_t^{(2,1)}$ as instrument for $f_t^{(6,6)}$
- Shorter-horizon forwards likely to be better measured: denser option strikes & more trading volume

Do Forward Rates Predict Future Spot Rates?

Instrumented Mincer–Zarnowitz Regressions for Spot Rates

$$\mu_{t+6}^{(6)} = \beta_0 + \beta_1 f_t^{(6,6)} + \epsilon_{t+6}, \quad f_t^{(6,6)} = \pi_0 + \pi_1 f_t^{(2,1)} + \eta_t$$

	(1) U.S.	**(2)** Ex-U.S.	**(3)** All
$f_t^{(6,6)}$	0.73***	0.69***	0.70***
	(0.062)	(0.078)	(0.074)
Intercept	0.59***		
	(0.13)		
Country FEs	✗	✓	✓
p-value: $\beta_1 = 1$	0.018	0.004	0.003
Obs.	378	1,849	2,227
R^2	0.22	0.20	0.22
Within R^2	—	0.13	0.14

SEs in (1) are heteroskedasticity and autocorrelation-robust [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Forward rate ↗ by **1%** $\Longrightarrow$ future spot rate ↗ by ∼**0.7%**
- Forward rates explain ∼**20%** of the variation in future spot rates
- The market **qualitatively** understands variation in the equity premium, but **quantitatively** significant excess persistence

Average Forecast Errors Are Close to Zero

Average Forecast Errors Across Countries

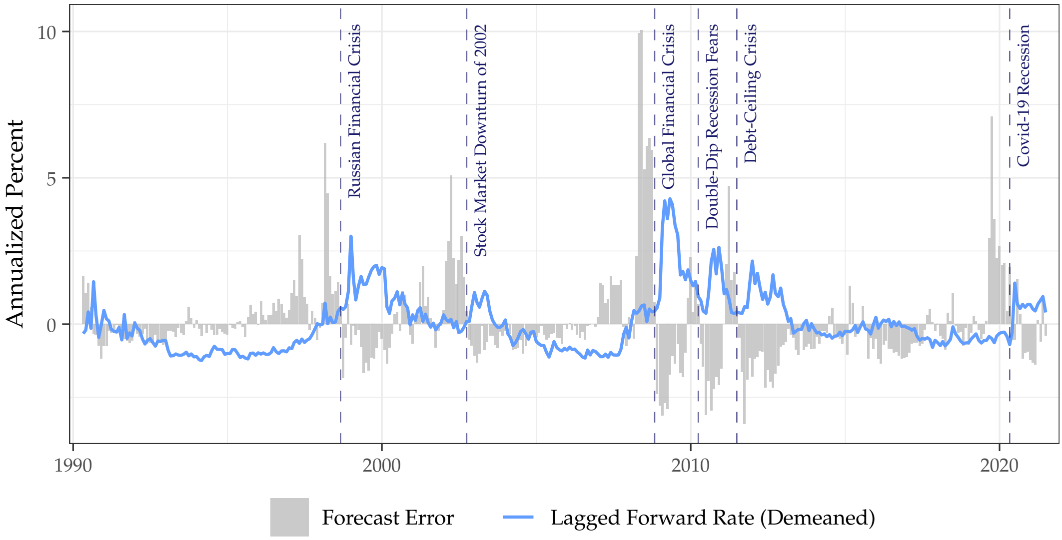
$$\varepsilon_{t+6}^{(6)} = \mu_{t+6}^{(6)} - f_t^{(6,6)}$$

	(1) U.S.	**(2)** Ex-U.S.	**(3)** All
Average	0.021	0.20	0.17
	(0.15)	(0.11)	(0.11)
Obs.	378	1,849	2,227

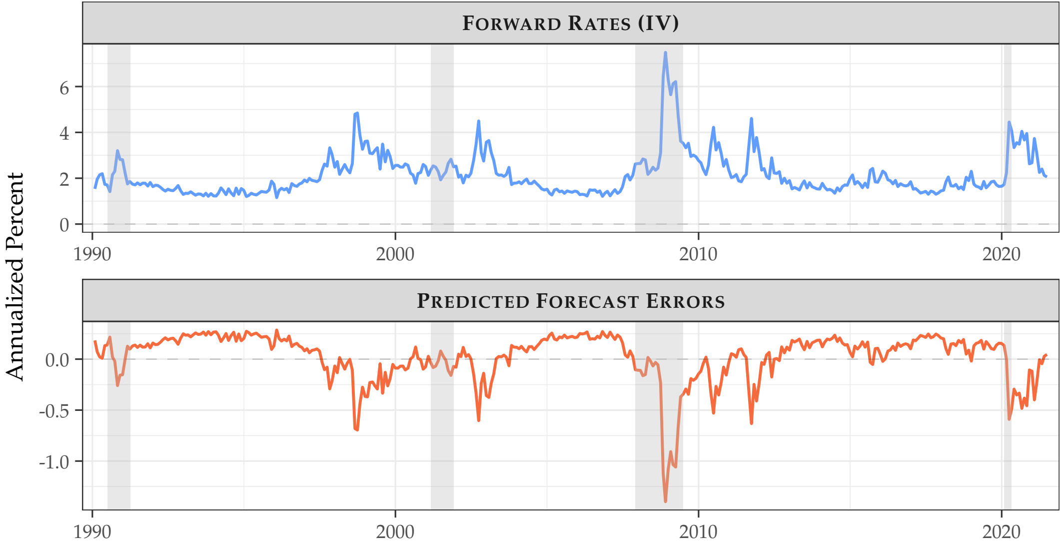
SEs in (1) are HAR [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Not just statistically insignificant, but effectively zero: $\bar{\varepsilon} \leqslant 20$ bps annualized
- Therefore can't reject log utility + RE just on the basis of average errors
  - Not the highest-powered test, but will be informative in trying to rationalize time variation
- But average of zero masks substantial predictability…

Forecast Errors and Lagged Forward Rates Over Time



Forward Rates as Predictors of Forecast Errors



Predictable Forecast Errors

Regressions of Forecast Errors on 2×1 Forward Rate

$$\varepsilon^{(6)}_{t+6} = \beta_0 + \beta_1 f^{(2,1)}_t + e_{t+6}$$

	(1) U.S.	**(2)** Ex-U.S.	**(3)** All
$f^{(2,1)}_t$	-0.17** (0.066)	-0.16** (0.049)	-0.16*** (0.047)
Intercept	0.39* (0.23)		
Country FEs	✗	✓	✓
Obs.	378	1,849	2,227
R^2	0.04	0.04	0.04
Within R^2	—	0.03	0.03

SEs in (1) are HAR [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

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Obs.	378	1,849	2,227
R^2	0.04	0.04	0.04
Within R^2	—	0.03	0.03

SEs in (1) are HAR [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Forward rates again overshoot future spot rates
- Errors are also predictable in Coibion–Gorodnichenko regressions using forward-rate *revisions*
- And predictability rises substantially ($R^2 = 0.11$) with kernel regression: Arises mostly from high forward rates
- Is this consistent with overreaction?
  It depends: Overreaction to what?
  - Option-based expected returns: Yes
    [Spot rates, fwd rates, fwd-rate revisions]
  - Past returns: Wrong direction
  - Consistent excess persistence

How Significant Are Forecast Errors?

Can now return to question posed at outset:
How significant are forecast errors for price variation?

$$p_t - d_t = \kappa - \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1} + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}$$

How Significant Are Forecast Errors?

Can now return to question posed at outset:
How significant are forecast errors for price variation?

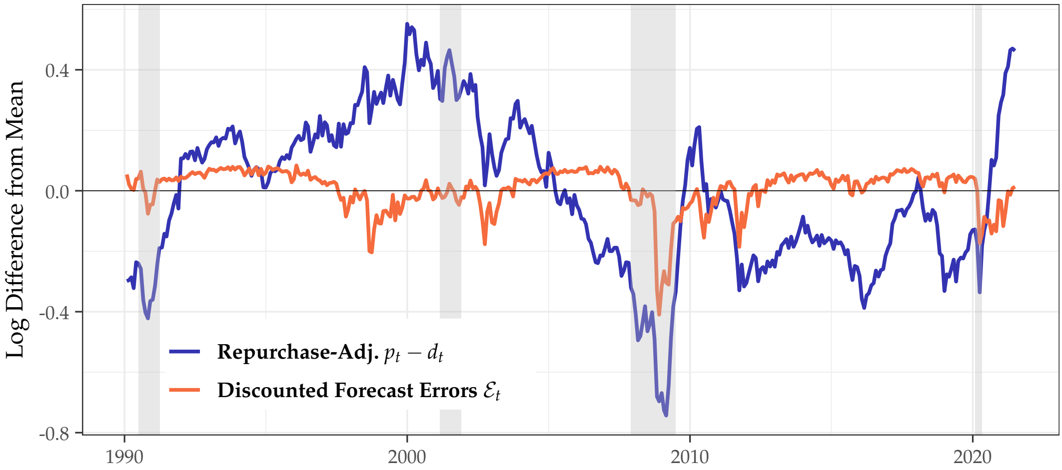
$$p_t - d_t = \kappa - \mathbb{E}_t r_{t+1} - \sum_{j=1}^{\infty} \rho^j f_t^{(j,1)} - \underbrace{RF_t}_{\substack{\text{discounted}\\ \text{risk-free rates}}} + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}$$

Break $f_t^{(j,1)}$ into:

$$f_t^{(j,1)} = \underbrace{\mathbb{E}_t[\mu_{t+j}^{(1)}]}_{\text{expected spot rates}} - \underbrace{\mathbb{E}_t[\varepsilon_{t+j}^{(1)}]}_{\text{predictable forecast errors}}$$

- Set one period to be 6 months, and predict error using 2m×1m forward
- Assume $\mathbb{E}_t[\varepsilon_{t+j+1}^{(1)}] = \phi^j \mathbb{E}_t[\varepsilon_{t+j}^{(1)}]$ [De la O & Myers (2021)] $\Longrightarrow \widehat{\phi} \approx 1$ *(using longer-dated SX5E data)*
- Use this to estimate contribution of discounted sum of predicted forecast errors ($\mathcal{E}_t$) on prices
- Compare to repurchase-adj. $p_t - d_t$ from Nagel & Xu (2022)

Discounted Forecast Errors and Price-Dividend Variation



Meaningful in magnitude, esp. in crises, and on average $\mathcal{E}_t$ moves 0.5-for-one with $p_t - d_t$

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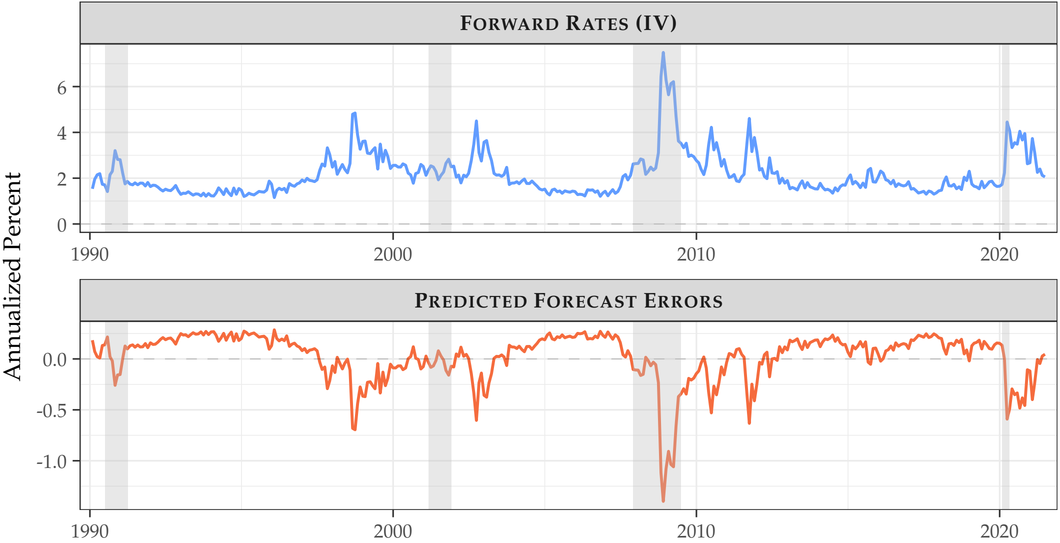
Data and Measurement: Surveys

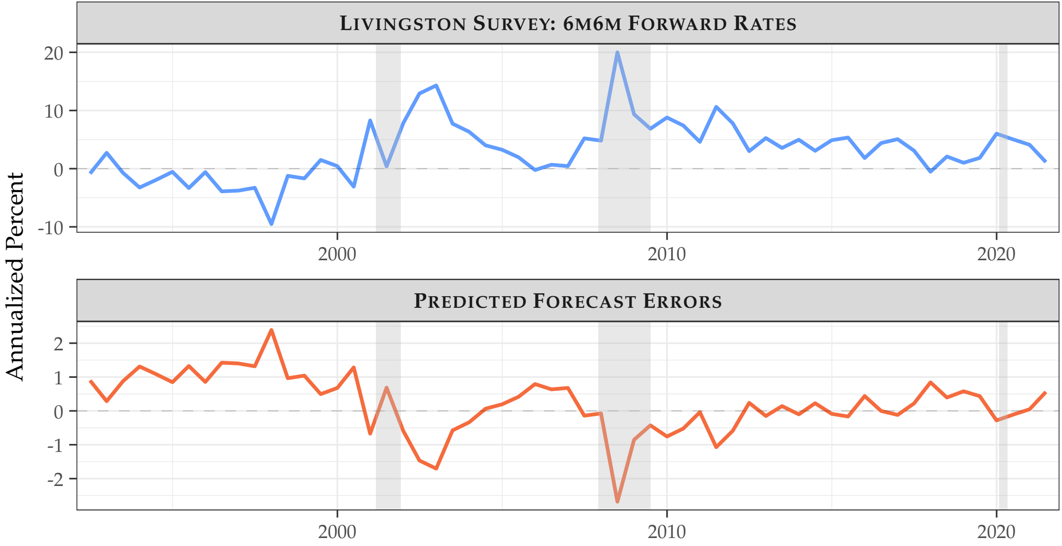
Survey Data:

- Livingston survey of prof. forecasters:
  - Price expectations at 6m & 12m horizon, allow for:
    - $\mu_t^{(12\text{ months})}, \mu_t^{(6\text{ months})}$
    - $f_t^{(6\text{ months})} = \mu_t^{(12\text{ months})} - \mu_t^{(6\text{ months})}$
    - $\varepsilon_{t+6\text{ months}}^{(6\text{ months})} = \mu_{t+6\text{ months}}^{(6\text{ months})} - f_t^{(6\text{ months})}$
- Duke CFO survey:
  - 1y & 10y return expectations, allow for:
    - $\mu_t^{(10\text{ years})}, \mu_t^{(1\text{ year})}$
    - $f_t^{(9\text{ years}, 1\text{ year})} = \mu_t^{(10\text{ years})} - \mu_t^{(1\text{ year})}$
    - $\varepsilon_{t+1\text{ year}}^{(9\text{ years})} \approx \mu_{t+1\text{ year}}^{(10\text{ years})} \times 9/10 - f_t^{(9\text{ years}, 1\text{ year})}$

Reminder: Forward Rates and Predicted Forecast Errors



Excess Cyclicality: Consistent Evidence in Surveys



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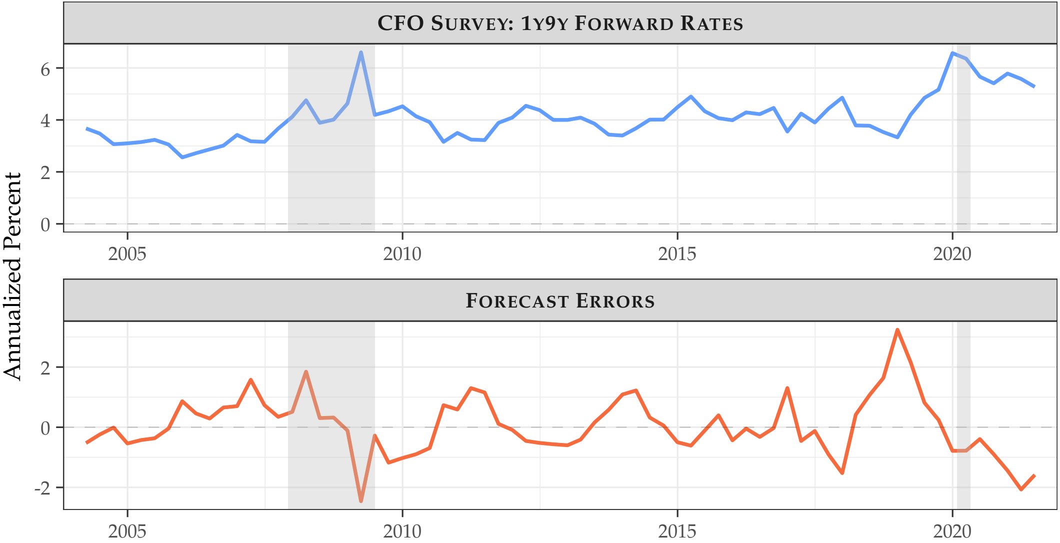


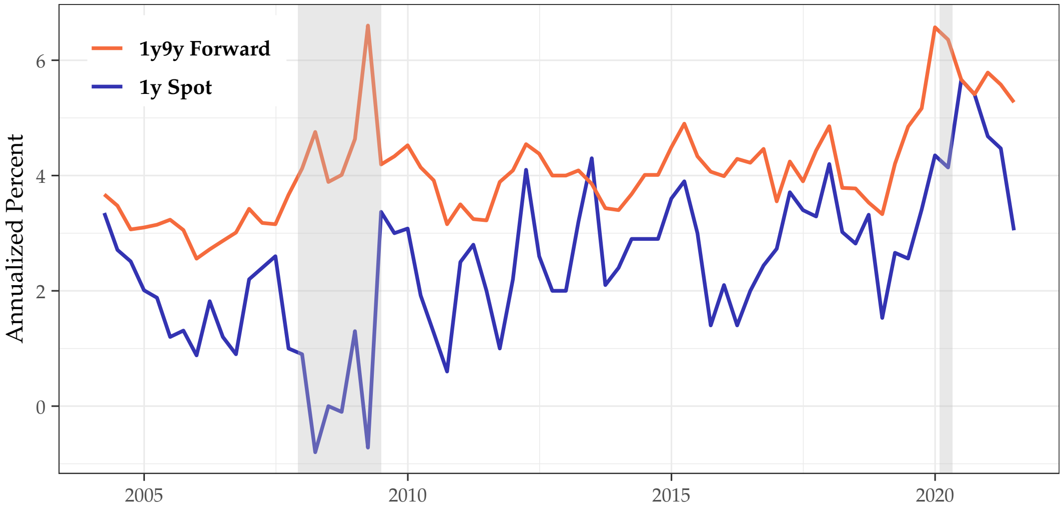
Predictive Regressions in Survey Data

	Expectations Measured by:		
	Options	Livingston Survey	CFO Survey
Panel A. Predictability in Spot Rates $(\mu_{i,t+1} = \beta_0 + \beta_1 f_{i,t} + e_{i,t+1})$			
β_1	0.88	0.68	0.63
R^2	0.71	0.38	0.46
Panel B. Predictability of Forecast Errors $(\epsilon_{i,t+1} = \beta_0 + \beta_1 f_{i,t} + e_{i,t+1})$			
β_1	-0.34	-0.19	-0.15
R^2	0.06	0.06	0.03

Consistency Across Measures

	Expectations Measured by:		
	Options	Livingston Survey	CFO Survey
Panel A. Correlation in Forward Rates Across Measures			
Options	1	0.46	0.11
Livingston Survey		1	0.55
Panel B. Cyclical Variation in Forward Rates and Forecast Errors			
$\rho\left(f_{i,t}, 1/\text{CAPE}_t\right)$	0.04	0.42	0.21
$\rho\left(\epsilon_{i,t+1}, 1/\text{CAPE}_t\right)$	-0.38	-0.19	-0.38

CFO Spot and Forward Rates: Importance of Long Horizon



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2. Price-Based Measurement of Expectations: Theory
3. Evidence from Price-Based Expectations
4. Evidence from Survey-Based Expectations
5. Explaining Forecast Errors
    - Can Forecast Errors From Price-Based Measure Be Rationalized?
    - A Model of Expectation Errors
6. Implications and Conclusions

Explaining Forecast Errors

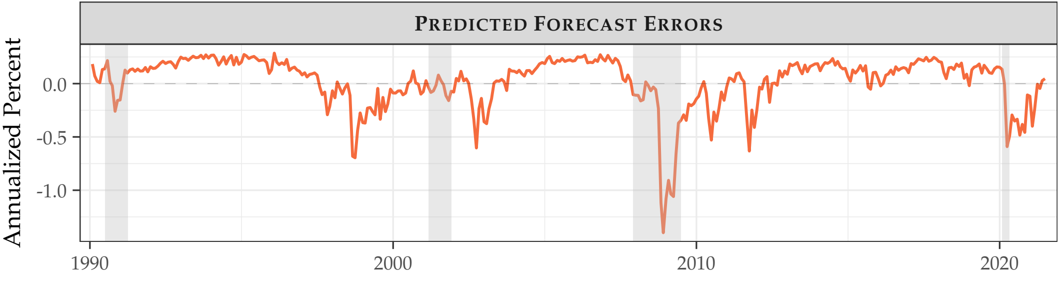
Paper considers two alternatives explaining forecast errors:

1. RE + risk premium
   - Price of discount-rate risk must be highly volatile and countercyclical for this to work
   - E.g., if $\text{Corr}_t(r_{t+1}, \mathbb{E}_{t+1} r_{t+2}) = -1$ and negative correlation condition (Gao & Martin 2021) $\implies$ **relevant SDF-related covariance can't change sign**
   - Doesn't work for survey evidence
2. Expectation errors
   - Simple calibrated model with log util. & diagnostic expectations
   - Increase in equity premium $\implies$ investors overestimate **future** equity premium
   - Single parameter θ governs overreaction to objective news
   - Consider range of values, incl. $\theta = 0$ [RE] & $\theta = 0.91$ [Bordalo et al. (2018, 2019) estimate]
   - θ around 0.9 does well at generating model coefficients close to our empirical estimates

Can Forecast Errors From Price-Based Measure Be Rationalized?

$$\mathbb{E}_t[\widehat{\varepsilon}^{(m)}_{t+n}] = \mathbb{E}_t[\varepsilon^{(m)}_{t+n}] - \underbrace{\text{Cov}_t(MR_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}])}_{\varsigma_t}$$

What conditions do we need on ς_t in order for **expectation errors** $\mathbb{E}_t[\varepsilon^{(m)}_{t+n}]$ to be unpredictable?
Must have $-\varsigma_t$ take same sign as pred. forecast errors:



Main challenge: Small on average, but must flip signs dramatically ($-$ in good times, $+$ in bad).

Can Forecast Errors From Price-Based Measure Be Rationalized?

$$\mathbb{E}_t[\widehat{\varepsilon}^{(m)}_{t+n}] = \mathbb{E}_t[\varepsilon^{(m)}_{t+n}] - \underbrace{\text{Cov}_t(MR_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}])}_{\varsigma_t}$$

What conditions do we need on ς_t in order for **expectation errors** $\mathbb{E}_t[\varepsilon^{(m)}_{t+n}]$ to be unpredictable?

- For simplicity: Take $n = m = 1$, and assume SDF and returns are jointly log-normal
- Then $\varsigma_t > 0$ (as needed in bad times) if and only if:

  $$SR_t(-\mu_{t+1}) > -\rho_t(r, \mu)\sigma_t(r),$$

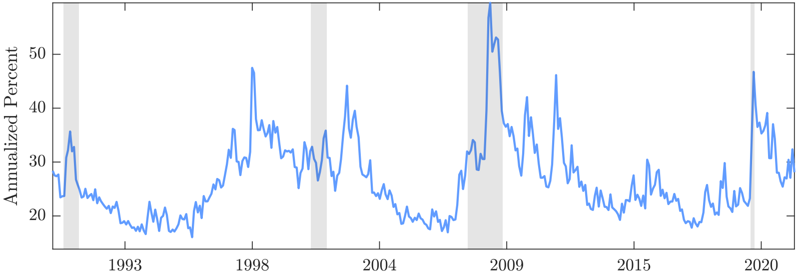
  where SR_t is Sharpe ratio on claim to next period's negative equity premium (low payoff is bad)
- Correlation $\rho_t(r, \mu)$ likely to be negative; for illustration, set it to -1
- Then SR_t must vary *more than* $\sigma_t(r)$ for ς_t to flip signs
- One calibration: Go back to log utility (likely to be conservative for time variation in σ_t), and estimate $\sigma_t(r)$ from options

Can Forecast Errors From Price-Based Measure Be Rationalized?

$$\mathbb{E}_t[\widehat{\varepsilon}_{t+n}^{(m)}] = \mathbb{E}_t[\varepsilon_{t+n}^{(m)}] - \underbrace{\text{Cov}_t(MR_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}])}_{\varsigma_t}$$

What conditions do we need on ς_t in order for **expectation errors** $\mathbb{E}_t[\varepsilon_{t+n}^{(m)}]$ to be unpredictable?

- SR_t must vary *more than* $\sigma_t(r)$ for ς_t to flip signs
- One calibration: Go back to conservative log utility case, and estimate $\sigma_t(r)$ from options.
  Results for conditional volatility of 6-month market return:



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What conditions do we need on ς_t in order for **expectation errors** $\mathbb{E}_t[\varepsilon_{t+n}^{(m)}]$ to be unpredictable?

- SR_t must vary *more than* $\sigma_t(r)$ for ς_t to flip signs
- Further, given $\rho_t(r, \mu) = -1$, ς_t **cannot** flip signs if mNCC [Gao & Martin (2021), Assumption 2] holds
  - $\rho_t(r, \mu) = -1 \implies \varsigma_t$ is scaled version of their covariance term $\mathcal{C}_t^{(n)}$
  - If $\mathcal{C}_t^{(n)} \leqslant 0$ (mNCC), then $\varsigma_t \leqslant 0$
- More generally, difficult to get both average errors (small) *and* time variation (large) right
- Paper has one illustration varying risk aversion γ

Model Setup

- Now want a simple lab to examine whether findings could plausibly arise from combo of:
  1. Log utility
  2. Expectation errors

⇒ consider a version of framework from Bordalo, Gennaioli, Shleifer (2018)

- 3-month spot rate dynamics under **objective** measure:

$$\mu_t^{(3)} = \left(1 - \sum_{j=1}^{3} \phi_j\right) \bar{\mu} + \phi_1 \, \mu_{t-1}^{(3)} + \phi_2 \, \mu_{t-2}^{(3)} + \phi_3 \, \mu_{t-3}^{(3)} + \epsilon_t, \qquad \epsilon_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$$

- **Under RE:** Term structure of current spot rates would be based on objective expectations $\mathbb{E}_t\left[\mu_{t+n}^{(3)}\right]$
- **Actual subjective expectations:** Excess sensitivity to news governed by "diagnosticity" parameter θ:

$$\mathbb{E}_t^{\theta}\left[\mu_{t+n}^{(3)}\right] = \mathbb{E}_t\left[\mu_{t+n}^{(3)}\right] + \theta \underbrace{\left(\mathbb{E}_t\left[\mu_{t+n}^{(3)}\right] - \mathbb{E}_{t-3}\left[\mu_{t+n}^{(3)}\right]\right)}_{\text{news} \propto \epsilon_t}$$

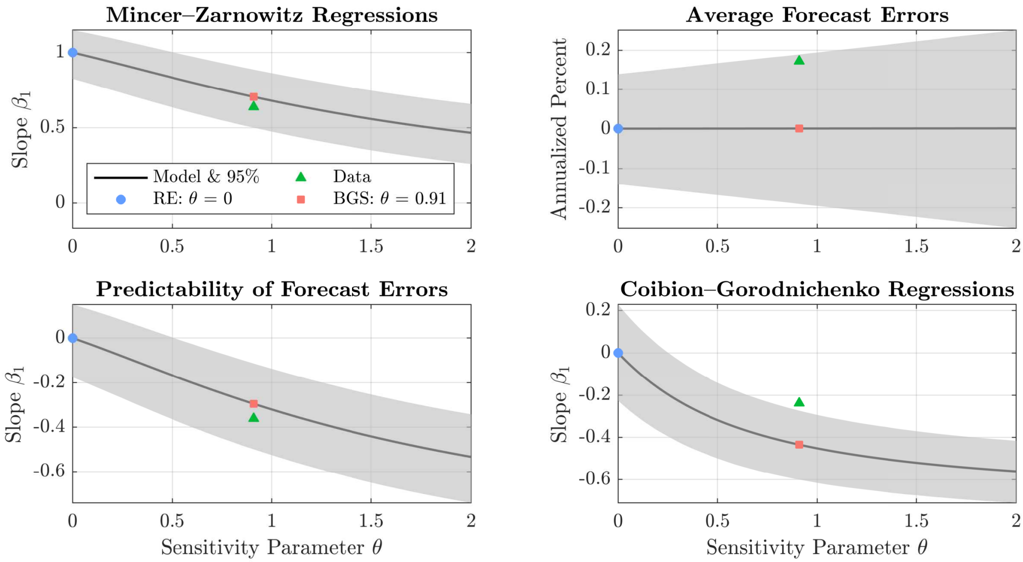
Model Setup

$$\mu_t^{(3)} = \left(1 - \sum_{j=1}^{3} \phi_j\right) \bar{\mu} + \phi_1\, \mu_{t-1}^{(3)} + \phi_2\, \mu_{t-2}^{(3)} + \phi_3\, \mu_{t-3}^{(3)} + \epsilon_t, \qquad \epsilon_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_\epsilon^2)$$

$$\mathbb{E}_t^\theta \left[\mu_{t+n}^{(3)}\right] = \mathbb{E}_t \left[\mu_{t+n}^{(3)}\right] + \theta \underbrace{\left(\mathbb{E}_t \left[\mu_{t+n}^{(3)}\right] - \mathbb{E}_{t-3} \left[\mu_{t+n}^{(3)}\right]\right)}_{\text{news} \propto \epsilon_t}$$

- Forward rates based on subjective expectations
- Longer-term spot rates embed objective short rate **and** subjective expectations of future short rates
- Consider a range of values for θ
  - $\theta = 0$: RE
  - $\theta = 0.91$: BGS (2018), BGLS (2019)
- Estimate objective parameters for spot-rate process in each country
- For each θ, simulate 10,000 samples and run same tests as in the data for $n, m = 6$ months

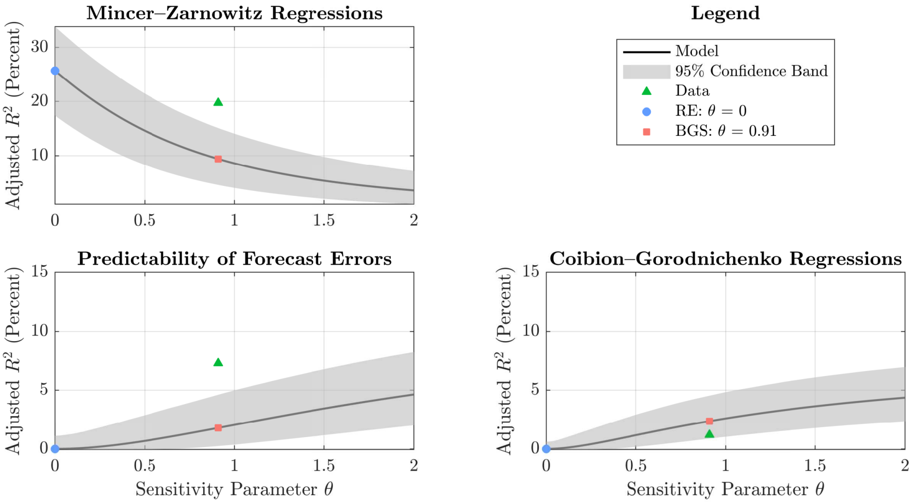
Model vs. Data: Main Estimates



Model vs. Data: R^2 Values

Simple calibration does reasonably well on main estimates...

...**but seems to miss some rational variation in forward rates:**



A Trilemma for Expectation Errors

More generally:

- While simple calibrated model does reasonably well at matching the data, again not an unqualified success for all possible notions of overreaction
- Subjective beliefs overreact to increases in *spot rates* in our model, not past returns, and cyclicality matters:

$$p_t - d_t = \kappa - \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1} \mathbb{E}_t r_{t+1} - \underbrace{\sum_{j=1}^{\infty} \rho^j f_t^{(j,1)}}_{\mathcal{F}_t} + \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}}_{CF_t} - RF_t$$

- Use $\tilde{\cdot}$ to denote **expectation error wedge** (deviation from RE economy):

$$\text{var}\left(\widetilde{p_t - d_t}\right) = \text{var}\left(\widetilde{\mathcal{F}}_t\right) + \text{var}\left(\widetilde{CF}_t\right) - 2\,\text{cov}\left(\widetilde{\mathcal{F}}_t, \widetilde{CF}_t\right)$$

- Have to choose between **two of three**:
  1. Volatile expectation errors for cash flows and/or returns
  2. Volatile price-dividend ratio relative to RE
  3. Positive comovement between fundamental and return expectation errors

Roadmap

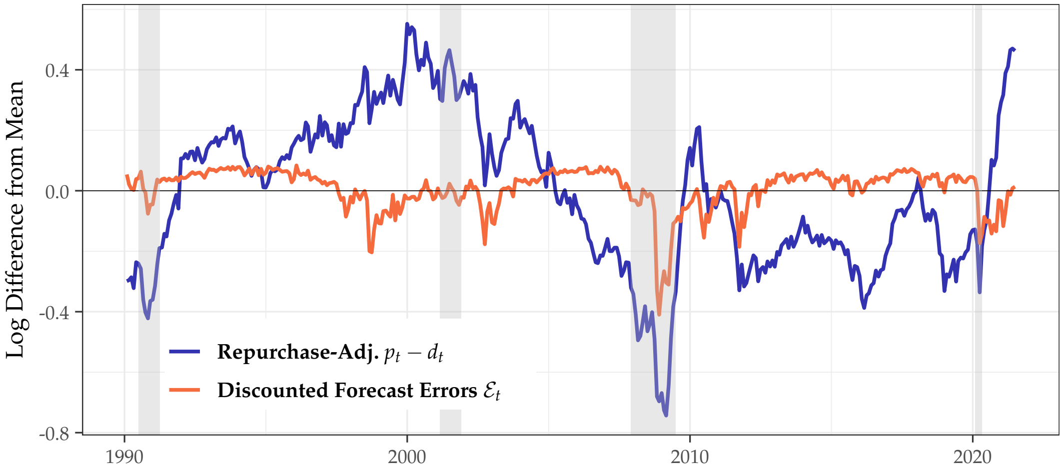
1. Introduction
2. Price-Based Measurement of Expectations: Theory
3. Evidence from Price-Based Expectations
4. Evidence from Survey-Based Expectations
5. Explaining Forecast Errors
6. Implications and Conclusions

Implications

Excess cyclicality in forward return expectations helps us understand:

1. Excess volatility in stock prices
   - When prices are depressed, this partly reflects investors expecting persistently high risk premia
   - If investors didn't overestimate persistence, would see more modest fluctuations in prices (about 50% less during 2008 crisis, nearly 100% less during Covid crash)
2. Inelastic demand for equities [Gabaix & Koijen 2022]
   - Puzzle: Why investors change weight in equities so modestly in response to change in stock prices
   - Partial resolution: If price drop leads to increases in expected returns **mainly at long horizons**, shouldn't see big increase in portfolio weight

Reminder: Forecast Errors and Price-Dividend Variation



Equity premium forecast errors help explain excess volatility, especially in crises.

Implications

Excess cyclicality in forward return expectations helps us understand:

3. Facts about equity term structure from dividend claims
   - Risk premia lower than expected $\Longrightarrow$ Realized returns higher than expected
   - Effect stronger for longer-duration assets (the market)
   - Potentially explains:
     - Downward sloping equity term structure on average [Binsbergen, Brandt, Koijen 2012]
     - Upward sloping term structure during bad times (counter-cyclical variation) [Gormsen 2021]
4. Debate on cyclicality of subjective risk premia
   - Short-term return expectations sometimes appear procylical [Greenwood & Shleifer 2014], acyclical [Nagel & Xu 2023], or countercyclical [Dahlquist & Ibert 2022]
   - Forward expectations are countercyclical across all data sources (and excessively so)
   - Disagreement in above studies may stem in part from differences in horizon

Final Notes

Summary:

- Introduce new methods to measure term structure of expected equity premia
- Robust evidence of excess countercyclicality in forward return expectations
- Investors consistently overestimate how long their own expected returns will stay elevated during bad times, and vice versa during good times
- Consistent across options (high-powered, general method) and surveys (straightforward measurement)

Tie-ins:

- Equity and bond term structures: Our tests are similar to tests of the expectations hypothesis, but with less room for discount-rate variation than in past work
- Similar to past work [van Binsbergen & Koijen (2017), Gormsen (2021)], find more predictability in equity term structure than in FI term structure
- Also build on Giglio & Kelly (2018) work on other term structures