Internet Appendix:

Forward Return Expectations *

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A Proofs

Proof of Proposition 1. Given that $M_{t,t+n}R_{t,t+n} = 1$ by assumption, $C_t^{(n)} = 0$ in (4). The stated results then follow immediately.

Proof of Proposition 2. First consider n = m = 1, and write

$$\varepsilon_{t+1}^{(1)} = \mu_{t+1}^{(1)} - f_t^{(1)} = \mathscr{L}_{t+1}^{(1)} - \mathscr{L}_t^{(2)} + \mathscr{L}_t^{(1)} + \operatorname{cov}_t(MR_{t,t+2}, r_{t,t+2}) - \operatorname{cov}_t(MR_{t,t+1}, r_{t,t+1}) - \operatorname{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2}).$$
(A1)

Consider the first covariance term. Given the joint log-normality of the SDF and returns (and the normality of $r_{t,t+n}$), Stein's lemma gives that

$$cov_t(MR_{t,t+2}, r_{t,t+2}) = cov_t(MR_{t,t+1}MR_{t+1,t+2}, r_{t,t+2})$$

= cov_t(mr_{t,t+1} + mr_{t+1,t+2}, r_{t,t+2}) \mathbb{E}_t[MR_{t,t+2}]
= cov_t(mr_{t,t+1} + mr_{t+1,t+2}, r_{t,t+1} + r_{t+1,t+2}),

where $mr_{t,t+n} = \ln(MR_{t,t+n})$, and where the last line uses that $\mathbb{E}_t[MR_{t,t+2}] = 1$. Having separated the two MR terms, apply Stein's lemma again to obtain

$$\operatorname{cov}_{t}(MR_{t,t+2}, r_{t,t+2}) = \operatorname{cov}_{t}(mr_{t,t+2}, r_{t,t+1}) + \operatorname{cov}_{t}(mr_{t,t+1}, r_{t+1,t+2}) + \operatorname{cov}_{t}(mr_{t+1,t+2}, r_{t,t+2})$$

=
$$\operatorname{cov}_{t}(MR_{t,t+2}, r_{t,t+1}) + \operatorname{cov}_{t}(MR_{t,t+1}, r_{t+1,t+2})$$

+
$$\operatorname{cov}_{t}(MR_{t+1,t+2}, r_{t+1,t+2}).$$
(A2)

For the first two terms in (A2), by the law of total covariance and using that $\mathbb{E}_{t+1}[MR_{t+1,t+2}] = 1$,

$$\operatorname{cov}_{t}(MR_{t,t+2}, r_{t,t+1}) = \mathbb{E}_{t} \left[MR_{t,t+1}r_{t,t+1} \operatorname{cov}_{t+1}(MR_{t+1,t+2}, 1) \right] + \operatorname{cov}_{t}(MR_{t,t+1} \mathbb{E}_{t+1}[MR_{t+1,t+2}], r_{t,t+1}) = \operatorname{cov}_{t}(MR_{t,t+1}, r_{t,t+1}),$$

$$\operatorname{cov}_{t}(MR_{t,t+1}, r_{t+1,t+2}) = \mathbb{E}_{t}[MR_{t,t+1} \operatorname{cov}_{t+1}(1, r_{t+1,t+2})] + \operatorname{cov}_{t}(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}]) = \operatorname{cov}_{t}(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}]).$$
(A3)

Turning now to the last term in (A1), the law of total covariance can similarly be applied to obtain that as of time t,

$$\mathbb{E}_t[\operatorname{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2})] = \operatorname{cov}_t(MR_{t+1,t+2}, r_{t+1,t+2}).$$
(A5)

Taking expectations in (A1), substituting in results (A2)–(A5), and applying the definition of $\hat{\varepsilon}_{t+1}^{(1)}$, we obtain:

$$\mathbb{E}_t \left[\varepsilon_{t+1}^{(1)} \right] = \mathbb{E}_t \left[\widehat{\varepsilon}_{t+1}^{(1)} \right] + \operatorname{cov}_t (MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}]) \,. \tag{A6}$$

Rearranging to solve for $\mathbb{E}_t[\widehat{\varepsilon}_{t+1}^{(1)}]$ yields the stated result for the n = m = 1 case. While this case is convenient for straightforward derivations, note that all the above steps apply when using t + n in place of t + 1 and using t + n + m in place of t + 2, so the stated result holds for general n, m. \Box

Proof of Proposition 3. Starting again with (A1) and expanding the first covariance term,

$$\operatorname{cov}_t(MR_{t,t+2}, r_{t,t+2}) = \operatorname{cov}_t(MR_{t,t+1}MR_{t+1,t+2}, r_{t,t+1}) + \operatorname{cov}_t(MR_{t,t+1}MR_{t+1,t+2}, r_{t+1,t+2}).$$
 (A7)

We consider each of the two terms on the right side of (A7) in turn, and in both cases apply the law of total covariance. For the first term, as in (A3),

$$\operatorname{cov}_t(MR_{t,t+1}MR_{t+1,t+2}, r_{t,t+1}) = \operatorname{cov}_t(MR_{t,t+1}, r_{t,t+1}).$$
(A8)

For the second term,

$$\operatorname{cov}_{t}(MR_{t,t+1}MR_{t+1,t+2}, r_{t,t+2}) = \mathbb{E}_{t}\left[MR_{t,t+1}\operatorname{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2})\right] + \operatorname{cov}_{t}(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}]).$$
(A9)

Using (A8) and (A9) in (A1), applying the definition of $\hat{\varepsilon}_{t+1}^{(1)}$, and taking expectations,

$$\mathbb{E}_{t}\left[\varepsilon_{t+1}^{(1)}\right] = \mathbb{E}_{t}\left[\widehat{\varepsilon}_{t+1}^{(1)}\right] + \mathbb{E}_{t}\left[(MR_{t,t+1}-1)\operatorname{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2})\right] \\ + \operatorname{cov}_{t}(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}]) \\ = \mathbb{E}_{t}\left[\widehat{\varepsilon}_{t+1}^{(1)}\right] + \operatorname{cov}_{t}(MR_{t,t+1}, \mathbb{E}_{t+1}[r_{t+1,t+2}] + \operatorname{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2})).$$
(A10)

Note from (4) that $\mathscr{L}_{t+1}^{(1)} = \mathbb{E}_{t+1}[r_{t+1,t+2}] + \operatorname{cov}_{t+1}(MR_{t+1,t+2}, r_{t+1,t+2})$. Using this in (A10),

$$\mathbb{E}_t \left[\widehat{\varepsilon}_{t+1}^{(1)} \right] = \mathbb{E}_t \left[\varepsilon_{t+1}^{(1)} \right] - \operatorname{cov}_t \left(M R_{t,t+1}, \mathcal{L}_{t+1}^{(1)} \right).$$

The above steps again apply when using t + n in place of t + 1 and using t + n + m in place of t + 2, completing the proof.

Proof of Lemma A1. To compute the risk-neutral expectation of $g(P_T) = R^{\alpha} (\ln R)^{\beta}$, we apply the Carr and Madan (1998) formula. Under standard regularity conditions, we have

$$\frac{1}{R_f} \mathbb{E}_t^* \Big[R^\alpha \left(\ln R \right)^\beta \Big] = \Big[g \Big(\bar{P} \Big) - \bar{P} g' \Big(\bar{P} \Big) \Big] \frac{1}{R_f} + g' \Big(\bar{P} \Big) P_t \\ + \int_0^{\bar{P}} g''(K) \operatorname{put}(K) dK + \int_{\bar{P}}^{\infty} g''(K) \operatorname{call}(K) dK$$

where $g'(P) \equiv \frac{\partial g}{\partial P}$ and $g''(P_T) \equiv \frac{\partial^2 g}{\partial P^2}$. The result follows by setting $\bar{P} = F_t^{(n)}$ and simplifying. \Box

Proof of Proposition A1. (A14) is immediate from Martin (2017) Result 8. (A15) and (A16) follow from Lemma A1 by setting the appropriate α and β and simplifying.

B Measurement Details

B.1 Options Data

U.S. Data. For the 1996 to 2021 period, we obtain end-of-day option prices, index prices, projected dividend yields, and risk-free rates from OptionMetrics. To maximize the sample size, we use options with non-standard expirations (weekly) and settlement (PM). We use the bid/ask midpoint as the option price in the main analysis. We linearly interpolate the risk-free rate curve to match option

maturities. If either the dividend yield or risk-free rate is missing, we use the last non-missing observation.

For the 1990 to 1995 period, we obtain intraday option prices from CBOE Market Data Replay, as in Culp, Nozawa, and Veronesi (2018). We obtain end-of-day index prices/returns from CRSP and estimate dividend yields from lagged one-year cum/ex-dividend index returns. We obtain Treasury bill rates and constant maturity Treasury yields from FRED to construct risk-free rates, again following Culp, Nozawa, and Veronesi.¹

Unlike OptionMetrics, CBOE provides intraday quotes. To construct end-of-day prices, we first apply filters to the intraday data and then use the last available quote. We drop quotes with the missing codes of 998 or 999. We drop quotes with negative bid-ask spreads. We correct erroneously recorded quotes — quotes with strike price less than 100 — by multiplying the strike/option price by 10. We drop end-of-day quotes that increase and then decrease fourfold (or vice versa), following similar filters in Andersen, Bondarenko, and Gonzalez-Perez (2015) and Duarte, Jones, and Wang (2024). We interpret these large reversals as probable data errors. To validate these filters, we compare data from CBOE and OptionMetrics in 1996. We match approximately 99.3% of option prices in OptionMetrics, suggesting these filters are not unreasonable.

We apply standard filters to the end-of-day data, as in Constantinides, Jackwerth, and Savov (2013), among others. (1) We drop options with special settlement. (2) To eliminate duplicate quotes, we select the quote with highest open interest. (3) We drop options with fewer than seven days-to-maturity. (4) We drop options with price less than 0.01. (5) We drop options with zero bid prices or negative bid-ask spreads. (6) We drop options that violate static no-arbitrage bounds:

$$\operatorname{put}(K) \le K e^{-r\tau} \qquad \operatorname{call}(K) \le P_t.$$

(7) We drop options for which the Black-Scholes implied volatility computation does not converge and options with implied volatility less than 5% or greater than 100%.

Global Data. We again obtain end-of-day option prices, index prices, projected dividend yields, and risk-free rates from OptionMetrics. Unlike the U.S. data, most option prices are either end-of-day settlement prices or last traded prices. Only a small fraction are from either bid/ask prices. The index price is time synchronized to the option price. If the index price is missing, we obtain the end-of-day price from Compustat Global. Risk-free rates are from currency-matched LIBOR curves. Dividend yields are from put-call parity and so are maturity-specific. As before with risk-free rates, we linearly interpolate the dividend yield curve to match option maturities. We apply the same filters to the end-of-day data as with the U.S. data, except for filters that require bid/ask prices.

Table A1 describes the global sample. The Europe and pan-Europe sample begins in January 2002 and ends in September 2021. The Asia-Pacific sample begins in January 2004 and ends in April 2021. Our global sample closely follows Kelly, Pástor, and Veronesi (2016) and Dew-Becker and Giglio (2023), but we also use pan-European Stoxx indexes. These indexes represent a substantive addition to the sample. At long maturities, the Euro Stoxx 50 is arguably the most liquid options market in the world, as is the case with the dividend futures market.²

Main Sample. As the global data is not equally robust across exchanges, we select the most reliable exchanges for the main analysis. We select the main sample by elimination. First, we drop Belgium, Finland, Netherlands, the Stoxx Europe 50/600, Korea, and Taiwan because they do not consistently

¹The CBOE website is https://datashop.cboe.com/mdr-quotes-trades-data.

²For more details about the dividend futures market, see Binsbergen and Koijen (2017).

have dense option surfaces, as seen in Panel A of Figure A9. Second, we drop Belgium, Korea, and Taiwan because they do not consistently have long-maturity options, as seen in Figure A10. Finally, we drop Finland, the Stoxx Europe 50/600, Sweden, China, and Japan because they do not consistently have deep out-of-the-money options, as seen in Figure A11. After dropping these exchanges, several for multiple reasons, we have 10 exchanges for the main sample, as seen in the last column of Table A1. As a robustness check, we examine the full sample of 20 exchanges in Table A8.

B.2 Option-Based Implementation

Methodology. On each date and separately for puts/calls,

- 1. We convert option prices to implied volatilities via Black-Scholes. We follow an extensive literature on option-implied risk-neutral densities that finds interpolation/extrapolation more conducive in the space of implied volatilities, not option prices.³
- 2. We fit a Delaunay triangulation to implied volatilities. The grid consists of strike prices between $\underline{K} = 0.10 \times P_t$ and $\overline{K} = 2.00 \times P_t$ with $\Delta K = 0.001 \times P_t$ and maturities $\tau = 30, 60,$ 91, 122, 152, 182, 273, 365 days. The triangulation extrapolates as necessary with the nearest implied volatility in simple moneyness and time-to-maturity space.⁴
- 3. We convert the triangulation of implied volatilities back to option prices via Black-Scholes. We then use the implied triangulation of option prices to evaluate the LVIX integral in (7) via Gaussian quadrature.
- 4. With the LVIX in hand, we can compute spot rates, forward rates, and forecast errors via Proposition 1, as visualized in Figure A7.
- 5. We occasionally find negative forward rates. Gao and Martin (2021) argue that negative forward rates are unlikely theoretically and likely represent data errors. We follow Gao and Martin and drop such observations, but our results are not quantitatively sensitive to this choice.

Discussion. Our methodology addresses the main empirical challenges in moment computation: discretization, truncation, and interpolation bias. Discretization and truncation bias relate to the strike dimension of option prices, interpolation bias to the maturity dimension. Our contribution is not to identify these biases — our discussion below closely follows that in Jiang and Tian (2007), among others — but rather to examine the implications for the LVIX.

First, discretization bias arises because (7) requires numerical integration. To address this bias, we integrate on a fine grid of interpolated option prices in step 3. Second, truncation bias arises because (7) requires integration over an infinite range of strikes in theory. In practice, we truncate the integral. To address this bias, we extrapolate and integrate over strikes well beyond the range of observable options in step 2. Finally, interpolation bias arises because (7) typically requires options with unavailable maturities. For example, we do not typically observe options with exactly 182 or 365 days-to-maturity. To address this bias, we interpolate the volatility surface at target maturities in step 2.

Our main concern is that the bias varies systematically. One possibility is cross-sectional variation: the bias is larger at longer maturities. If we overestimate long-maturity forward rates, then we would

³For reviews of the literature on risk-neutral densities, see Figlewski (2010) and Malz (2014).

⁴We implement the Delauney triangulation in Matlab via the scatteredInterpolant function.

find more negative forecast errors. Another possibility is business-cycle variation: the bias is larger in bad times. If we overestimate forward rates in bad times, then we would find more predictable forecast errors. In either case, measurement error works against the rational expectations null.

We examine the role of measurement error below.

C Measurement Error

C.1 Alternative Integration Bounds

Theory. We first analyze truncation bias in theory. To motivate the analysis, we note that the integrand in (7) — the option price divided by the strike price — approaches zero in both integration limits:

$$\lim_{K \to 0} \frac{\operatorname{put}(K)}{K} = 0 \qquad \lim_{K \to \infty} \frac{\operatorname{call}(K)}{K} = 0.$$

This implies that *all* of the integral's mass corresponds to observable, near-the-money strikes and *none* corresponds to unobservable, deep out-of-the-money strikes. Figure A8 visualizes this point in the Black-Scholes model.

Our point above is qualitative: truncation bias might not be large and might even be small. To understand the quantitative implications, we turn to simulations of parametric option pricing models. With knowledge of the true data generating process, we can quantify how methodological choices — truncation of the integral and extrapolation of the implied volatility surface — affect truncation bias. We focus on how the bias varies between short and long maturities as well as good and bad times. Our thought experiment closely follows a similar exercise for the VIX in Jiang and Tian (2007).

Figure A1 reports the results. Panel A truncates the volatility surface. As an example, the 20% truncation bound evaluates the integral from strike $\underline{K} = 0.80 \times P$ to $\overline{K} = 1.20 \times P$. On the left, we consider a Black-Scholes model. We find a small truncation bias in good times but a large truncation bias in bad times. In good times, volatility is low, deep out-of-the-money options are cheap, and so the bias is small. In contrast, in bad times, volatility is high, deep out-of-the-money options are expensive, and so the bias is large. The bias in forward rates is especially large because longer-maturity option prices have more time value. On the right, we consider a stochastic volatility model with jumps (SVJ). We again find an uncomfortably large truncation bias. Relative to Black-Scholes, the bias is larger when volatility is low, but smaller when volatility is high because volatility mean reverts in SVJ.⁵

Panel B extrapolates the volatility surface. As an example, the 20% extrapolation bound first truncates the volatility surface at the 20% out-of-the-money strike, then extrapolates the surface with the corresponding implied volatility, and finally evaluates the integral from strike $\underline{K} = 0.10 \times P$ to $\overline{K} = 2.00 \times P$. We continue with a SVJ model. Relative to Panel A, we find that extrapolation reduces truncation bias across the board. More importantly, truncation bias is small in both good and bad times. Our extrapolation scheme reduces, if not eliminates, the cyclical component in truncation bias.

To build intuition as to why extrapolation works, we formally describe the experiment. Let $put(K, \sigma)$ be the Black-Scholes price of a put with strike K and implied volatility σ . Let $IV_p(K)$ be the true implied volatility of a put with strike K. In the left tail, the experiment replaces true put prices with the approximation:

 $\operatorname{put}(K, IV_p(K)) \approx \operatorname{put}(K, IV_p(0.80 \times P))$ for $K \leq 0.80 \times P$.

⁵We compute SVJ prices in Matlab via the optByBatesNI function.

The notation and approximation are analogous for calls. In the right tail, the experiment replaces true call prices with the approximation:

$$\operatorname{call}(K, IV_c(K)) \approx \operatorname{call}(K, IV_c(1.20 \times P)) \text{ for } K \ge 1.20 \times P.$$

The success of this approximation depends on two factors. This first is the wedge between near-themoney and deep out-of-the-money volatility. The second is the sensitivity of Black-Scholes prices to volatility. This sensitivity approaches zero in both integration limits:

$$\lim_{K \to 0} \frac{\partial \operatorname{put}(K, \sigma)}{\partial \sigma} = 0 \qquad \lim_{K \to \infty} \frac{\partial \operatorname{call}(K, \sigma)}{\partial \sigma} = 0$$

This implies that what matters is not *how* we extrapolate but rather *whether* we extrapolate; that is, the wedge is second-order.

The analysis above suggests that truncation bias leads to underestimates of forward rates, especially in bad times, but that extrapolation can reduce this bias, especially its cyclical component. The analysis is somewhat silent on forecast errors because we did not take strong stand on the dynamics on the underlying state variable. To make progress on this front, we must turn to the data, which we do so below.

Data. We next analyze truncation bias in the data. To motivate the analysis, we note that there is business-cycle variation in the integral's support. In bad times, stock prices decrease, and this shifts the distribution of strikes upward. Figure A2 visualizes this point in the U.S. sample. This suggests that truncation bias might be able to generate systematic measurement error, which is required for measurement error to explain forecast-error predictability.⁶

To better understand the relevance of this threat, Table A3 examines alternative integration bounds. To build intuition, the top subpanel artificially truncates the volatility surface. The first row truncates the volatility surface at the 25% out-of-the-money strike. Let K_{min} be the minimum put strike in the data. The lower integration bound is

$$\underline{K} = \max\left\{K_{min}, 0.75 \times P_t\right\}.$$

The notation and bound are analogous for calls. Let K_{max} be the maximum call strike in the data. The upper integration bound is

$$\overline{K} = \min\left\{K_{max}, 1.25 \times P_t\right\}.$$

In other words, at best, the first row evaluates the integral in (7) from $\underline{K} = 0.75 \times P_t$ and $\overline{K} = 1.25 \times P_t$, but the bounds might be even shallower, depending on the availability of option prices. The next four rows consider deeper but otherwise analogous bounds. The sixth row evaluates the integral over all observable strikes. Each of these bounds naturally varies both by time and maturity with the availability of option prices.

When considering very narrow truncation limits, our results for risk premia are statistically less strong, which is somewhat expected given that this induces severe truncation bias in the approximated integral. More generally, with shallow bounds, forecast errors are relatively large on average but less predictable. With deep bounds, forecast errors are relatively small on average but more predictable. The fact that our results become weaker following the artificial truncation

⁶In contrast, neither discretization nor interpolation bias varies with the business cycle. The spacing of option strikes is typically time-invariant. The availability of option maturities typically varies deterministically.

suggests that the limited availability of strikes does not mechanically produce our results – in fact, such limitations in the data would weaken rather than strengthen our results.

As mentioned above, for measurement error (which includes, but is not limited to, truncation bias) to explain the results, we would have to *overestimate* forward rates in bad times. This upward bias would mechanically generate predictably negative forecast errors. But the experiment above suggests that in the presence of measurement error driven bias truncation bias, we would likely *underestimate* forward rates in bad times. Figure A3 visualizes this point by showing how the estimated forward and spot risk premia change under the different integration bounds. Measurement error therefore does not seem to help explain the results, either quantitatively as in the table or even qualitatively, as it works in the wrong direction.

The bottom subpanel extrapolates the volatility surface. The first row considers static integration bounds, following a similar robustness check in Gormsen and Jensen (2023):

$$\left[\underline{K}^{(n)}, \overline{K}^{(n)}\right] = \begin{cases} [0.75, 1, 25] \times P_t & n \in \{1, 2\}\\ [0.55, 1.45] \times P_t & n \in \{3, 4, 5\}\\ [0.35, 1.65] \times P_t & n \in \{6, 9\}\\ [0.20, 1.80] \times P_t & n \in \{12\} \end{cases}.$$

These bounds vary by maturity. This variation is logical because deep out-of-the-money options have more time value at longer maturities. The second row considers dynamic integration bounds, again following a similar robustness check in Gormsen and Jensen:

$$\underline{K}^{(n)} = \max\left\{0.10, 1.00 - 5\sigma_t^{(n)}\sqrt{\tau}\right\} \times P_t \qquad \overline{K}^{(n)} = \min\left\{2.00, 1.00 + 5\sigma_t^{(n)}\sqrt{\tau}\right\} \times P_t,$$

where $\sigma_t^{(n)}$, the price of the volatility contract in Bakshi, Kapadia, and Madan (2003), proxies for market volatility:

$$\left(\sigma_t^{(n)}\sqrt{\tau}\right)^2 = \frac{1}{R_f} \mathbb{E}_t^* \left[(\ln R)^2 \right]$$

$$= \int_0^{P_t} \frac{2\left(1 + \ln\left[\frac{P_t}{K}\right]\right)}{K^2} \operatorname{put}(K) dK + \int_{P_t}^\infty \frac{2\left(1 + \ln\left[\frac{P_t}{K}\right]\right)}{K^2} \operatorname{call}(K) dK.$$
(A11)

These bounds vary by time with volatility. This variation is logical because deep out-of-the-money options are more expensive in bad times. The third row considers the baseline integration bounds, as discussed in Appendix B.2. Regression slopes and average errors are quantitatively consistent across all extrapolation schemes, consistent with the idea above that what matters is not how we extrapolate but rather whether we extrapolate.

C.2 Alternative Measures

The appendix considers robustness checks where we use alternative choices to measure spot rates, forward rates, and forecast errors. Some of these choices are motivated by liquidity. As an example, if liquidity were an issue, then we would expect use of bid/ask prices to significantly alter the results (this is not the case). Other choices are motivated as checks on how exactly we fit a volatility surface to the data. As an example, the SVI surface below explicitly imposes no-arbitrage on the surface (which does not seem to matter either).

Table A4 presents the results. Regression slopes and average errors are quantitatively robust to

these choices in all cases, while maintaining their statistical significance in most cases.

Liquidity Filters. The top subpanel repeats the analysis with additional liquidity filters. The second row considers an outlier filter, following similar filters in Constantinides, Jackwerth, and Savov (2013) and Beason and Schreindorfer (2022). On each date and separately for puts/calls, we first fit a quadratic function to implied volatilities in terms of moneyness K/P and time-to-maturity. To minimize the effect of deep out-of-the-money, short/long-maturity options, we only use options with moneyness $0.65 \leq K/P \leq 1.35$ and maturity $14 \leq \tau \leq 365$ days. We then drop influential observations via Cook's Distance. The third row considers an open interest filter. We drop options with zero open interest, as seen in Panel B of Figure A9. We do not have open interest data before 1996. The fourth row combines the outlier and open interest filters.

Bid/Ask Prices. The bottom subpanel repeats the analysis with bid/ask prices, following similar robustness checks in Martin (2017) and Gao and Martin (2021). We only have bid/ask prices in the U.S. sample. The first row reports the baseline results with the bid-ask midpoint. The second row repeats the analysis with bid prices, the third ask prices.

Volatility Surface. The second-to-last row in the top subpanel repeats the analysis with the interpolated volatility surface from OptionMetrics. OptionMetrics provides interpolated Black-Scholes implied volatilities on a constant moneyness/maturity grid.⁷

SVI Surface. The last row in the top subpanel repeats the analysis with the stochastic volatility inspired (SVI) surface from Jim Gatheral at Merrill Lynch.⁸ Our implementation of the SVI surface closely follows Berger, Dew-Becker, and Giglio (2020) and Beason and Schreindorfer (2022). We parameterize squared Black-Scholes implied volatilities with the function

$$\sigma_{BS}^{2}(t,\kappa,\tau) = a + b\left(\rho\left(\kappa - m\right) + \sqrt{\left(\kappa - m\right)^{2} + \sigma^{2}}\right),\tag{A12}$$

where κ is standardized forward moneyness

$$\kappa = \frac{\ln K - \ln F_t^{(n)}}{\sigma_t^{(n)} \sqrt{\tau}},$$

 $\sigma_t^{(n)}$ proxies for the risk-neutral volatility of the market return as in (A11), and each parameter is a linear function of time-to-maturity (e.g., $a = a_0 + a_1 \tau$).

On each date, we estimate parameters $\theta = (a_0, a_1, b_0, b_1, \rho_0, \rho_1, m_0, m_1, \sigma_0, \sigma_1)$ that minimize the implied volatility RMSE between the surface (A12) and the data, subject to standard no-arbitrage constraints: option prices are nonnegative and monotonic/convex in K. We check these constraints on a grid with moneyness between $-20 \le \kappa \le 0.50$ for puts, between $-0.50 \le \kappa \le 10$ for calls, and maturities $\tau = 30, 60, 91, 122, 152, 182, 273, 365$ days. We estimate the surface with outlier-filtered, as discussed above, out-of-the-money puts/calls: puts with $\kappa \le 0$ and calls with $\kappa \ge 0$. We estimate the surface separately for puts/calls and separately for short/long-maturity options: $14 \le \tau \le 122$

⁷The literature often uses this surface for American-exercise options because OptionMetrics reports an equivalent, European-exercise, implied volatility (see, for example, Kelly, Lustig, and Van Nieuwerburgh 2016 and Martin and Wagner 2019).

⁸For more details, see Gatheral and Jacquier (2011, 2014). For a textbook treatment, see Gatheral (2011).

days and $122 < \tau \leq 365$ days, respectively.⁹

C.3 Additional Robustness Checks

This appendix examines additional empirical exercises related to measurement error. We exploit cross-sectional variation in liquidity across countries and across horizons as well as time-series variation in liquidity in the U.S. sample. We discuss each in turn.

This empirical evidence shows that liquidity-driven measurement error, in fact, weakens our results. This is because illiquidity biases our estimates of forward rates downwards, not upwards as required for measurement error to generate spurious results. This evidence makes us confident that our results on forward rates and forecast errors are driven by the true underlying process of these variables and are very unlikely to be an artifact of measurement error.

Split-Sample Regressions. The first piece of evidence comes from using time variation in the liquidity of option markets in the U.S. sample. Option markets have generally become more liquid over time — on any given trading day, there are literally thousands more options in 2021 than in 1990. More concretely, the surface is more dense, as seen in Figure A12, and the surface extends further into the tails, as seen in Figure A2.

Table A5 reports the results. In the first half, when measurement error is more severe, forecast errors are less predictable. In the second half, when measurement error is less severe, forecast errors are more predictable, even without NBER recessions. That is, our results are stronger in the latter part of our sample. While things other than liquidity have also changed over time, this result similarly suggests that measurement error biases us against our result, and that observing a complete and more liquid set of option prices would strengthen our results.

Alternative Horizons. The next piece of evidence comes from using cross-sectional variation in liquidity across horizons. Short-maturity options (less than 3 months) are substantially more liquid than long-maturity options (more than 3 months), not only in the U.S. sample but also worldwide.

Table A6 considers short horizons: $2 \le n + m \le 3$. At these horizons, measurement error is less of a concern. The surface is more dense and extends further into the tails, as seen in Figure A12 and A2. Moreover, the assumptions that the expectations hypothesis holds (see Section 2.2 for more details) and that dividends are known ex ante (see Section 3.1.2 for more details) are more reasonable at short horizons. Our results continue to hold at shorter horizons. This finding reinforces the idea that measurement error can only weaken our results.

Table A7 considers longer horizons: $4 \le n + m \le 12$ (the baseline horizon is n = m = 6 in the bottom subpanel). At these horizons, measurement error remains a concern. That said, the results continue to hold at these horizons. Holding n + m fixed, forecast errors are relatively small on average and less predictable with small n, relatively large on average and more predictable with large n.

Alternative Samples. The last piece of evidence arises from leveraging cross-country differences in liquidity in the options market. Some countries in our sample have relatively illiquid options markets (like Australia or Spain), while others have very liquid markets (like Germany, the Euro Stoxx 50, and of course, the U.S.). This is especially true when considering the main sample of 10 exchanges versus the full sample of 20 exchanges, as shown in Table A1, which were explicitly categorized on the basis of liquidity. We show that our results are stronger in the most liquid option markets. This

⁹For more details about no-arbitrage violations, see Aït-Sahalia and Duarte (2003).

finding suggests that if measurement error influences our results, it does so by *weakening* our results and biases us towards finding no effects.

As motivation, Figure A13 visualizes the relationship between spot and forward rates in the global sample, analogous to Figure 2 in the U.S. sample. Table A8 reports the corresponding results. Given liquidity concerns, we make two important changes here relative to the baseline analysis. First, we focus on horizons substantively shorter than the baseline analysis because, as discussed above, there is even less liquidity at long horizons in the non-U.S. sample and especially in the less-developed option markets considered in this table. So, while liquidity remains a concern in the less-developed markets, it is probably less slightly severe at shorter horizons. Second, we integrate over only observable option prices (that is, we do not extrapolate the surface into the tails as in the baseline analysis). We did not want to spuriously extrapolate the surface and the analysis above in which we consider alternative integration bounds suggests that this is likely to be a conservative approach (that is, bias the results towards the rational expectations null). The table reports results that are quantitatively similar to, but at times somewhat weaker than, that in the main sample. That said, the fact that the results are not stronger in the full sample pushes against the notion that illiquidity in itself can produce rejections of rationality.

As an additional test, we also find that the slopes are substantively weaker — that is, closer to rational expectations null — in exchanges with more illiquid markets, further suggesting that liquidity-driven measurement error cannot spuriously produce significant results. Unlike the full sample analysis above, we conduct this analysis within the main sample and for the baseline horizon, so this likely speaks better to results as a whole. We briefly outline this additional but unreported robustness check. Our proxy for option liquidity is the share of long-maturity options with positive open interest. The cross-exchange ordering of liquidity is reasonable. The usual suspects, like the SPX, SX5E, and DAX, are the most liquid. With this proxy, we examine the cross-exchange relationship between liquidity and the Mincer-Zarnowitz and error-predictability regression slopes. For both slopes, we find a strong negative correlation. Our takeaway, once again, is that that illiquidity works against the results: exchanges with less liquidity have slopes closer to the rational expectations null.

D Additional Empirical Results

D.1 Power Utility Regressions

This appendix derives the power utility analogue to the LVIX in Section 4.1.5. To do so, we apply results from Breeden and Litzenberger (1978) and Martin (2017). We omit time and time-to-maturity dependence throughout for simplicity.

Lemma A1 (Spanning). For any α and β ,

$$\frac{1}{R_f} \mathbb{E}_t^* \Big[R^\alpha \left(\ln R \right)^\beta \Big] = R_f^{\alpha - 1} \left(\ln R_f \right)^\beta + \int_0^F \omega(\alpha, \beta) \operatorname{put}(K) dK + \int_F^\infty \omega(\alpha, \beta) \operatorname{call}(K) dK,$$

where

$$\omega(\alpha,\beta) = -\frac{\alpha \left(1-\alpha\right) m^{\beta} + \beta \left(1-2\alpha\right) m^{\beta-1} + \beta \left(1-\beta\right) m^{\beta-2}}{P_t^2} \left(\frac{K}{P_t}\right)^{\alpha-2}$$
(A13)

and $m = \ln K - \ln P_t$.

As is well-known, under standard regularity conditions, we can compute the price of any function

of the index price via a replicating portfolio of bonds, stocks, and options. We simply apply this result to the function $R^{\alpha} (\ln R)^{\beta}$, which is useful for power utility expectations below.¹⁰

Proposition A1 (Power Utility). From the standpoint of an unconstrained power utility investor fully invested in the market,

$$\mathbb{E}_t[\ln R] - \ln R_f = \frac{\mathbb{E}_t^*[R^\gamma \ln R]}{\mathbb{E}_t^*[R^\gamma]} - \ln R_f,$$
(A14)

where

$$\frac{1}{R_f} \mathbb{E}_t^*[R^\gamma \ln R] = R_f^{\gamma-1} \ln R_f + \int_0^F \omega(\gamma, 1) \operatorname{put}(K) dK + \int_F^\infty \omega(\gamma, 1) \operatorname{call}(K) dK,$$
(A15)

and

$$\frac{1}{R_f} \mathbb{E}_t^*[R^{\gamma}] = R_f^{\gamma-1} + \int_0^F \omega(\gamma, 0) \operatorname{put}(K) dK + \int_F^\infty \omega(\gamma, 0) \operatorname{call}(K) dK,$$
(A16)

and γ is the investor's risk aversion.

The LVIX is a special case of (A14) with $\gamma = 1$, and so the mechanics under power utility are similar, if only messier, to that under log utility. However, there is one caveat: as risk aversion γ increases, the weights on deep out-of-the-money call options in (A13) become untenably large. Since these options are largely unobservable, we can only realistically measure expectations for a $\gamma \leq 3$ investor in practice.

Armed with equity premia from the standpoint of a power utility investor, we can compute spot rates, forward rates, and forecast errors in the usual way. Table A9 reports regression slopes and average errors for different values of risk aversion. Figure A14 visualizes the results. See Section 4.1.5 for a more thorough discussion.

D.2 Long-Horizon Forecast Errors

This appendix provides details on the forecast-error quantification in Section 5.1.

For the quantification, we re-estimate spot rates, forward rates, and forecast errors at longer horizons (up to m + n = 8 years) for the Euro Stoxx 50 (SX5E). The sample runs from September 2005 through September 2014 (beyond which we cannot yet observe realized forecast errors). The combinations of m and n (in months) can be seen in Table A10.

We are only able to do this analysis for Euro Stoxx 50. No other exchange has comparably long-maturity options available for such a long period. The only other exchange with such long maturities is the FTSE 100, but this sample only starts in 2014. More importantly, long-maturity options are liquid for the SX5E: the share of contracts with positive open interest is very consistent for medium (1 to 3 years) and long (3 to 10 years) maturities. In contrast, for the FTSE 100, there is a severe decline in the relative liquidity of long-maturity options (that is, almost no liquidity as per open interest). A few exchanges in the main sample do have option with maturities up to 5 years, but again, the liquidity of these options is suspect, rendering them unusable, at least for this analysis.

¹⁰Lemma A1 clarifies the links among several volatility indexes. The VIX index from the CBOE is a function of $\mathbb{E}_t^*[\ln R]$, the SVIX index from Martin (2017) a function of $\mathbb{E}_t^*[R^2]$, and the LVIX index from Gao and Martin (2021) a function of $\mathbb{E}_t^*[R \ln R]$.

For each such combination of m and n, we predict forecast errors as in (9) using a regression of realized forecast errors on shorter-horizon forward rates; we use the $n - 12 \times 12$ forward rate (with horizons again now in months) for $n \ge 24$, and for n = 12 we use the 6×6 rate. After obtaining these predicted forecast errors, we calculate a decay parameter for each date's forecast errors, $\phi_t^{(n,m)}$, as the ratio of estimated $\mathbb{E}_t[\tilde{\varepsilon}_{t+n}^{(m)}]$ to $\mathbb{E}_t[\tilde{\varepsilon}_{t+12}^{(12)}]$ for each available m, n > 12. This decay specification builds on the one used by De Ia O and Myers (2021, eq. (13)) for expected returns, but we estimate it directly for each date t (whereas they use a full-sample regression for one horizon).

The entries of Table A10 report the median decay parameter over all t for each combination of m and n. In all cases the estimates are close to or above 1. Assuming that predictable forecast errors are permanent at all horizons might be thought of as providing an estimate of their maximal possible effect. That said, when we estimate the decay parameter in the U.S. (at shorter horizons, unreported), we in fact generally obtain estimates greater than 1, suggesting that setting $\phi^{(n,m)} = 1$ may, if anything, be slightly conservative in the U.S. sample.

E Model Details

E.1 Calibration Implementation

Fundamentals. We measure fundamentals with option-implied volatility (variance, not standard deviation). We omit time and horizon dependence below for simplicity. From the standpoint of an unconstrained log utility investor fully invested in the market, the conditional variance of the 3-month market return is:

$$x_t \equiv \sigma_t^2 \left(\ln R \right) = \mathbb{E}_t \left[(\ln R)^2 \right] - \left(\mathbb{E}_t \left[\ln R \right] \right)^2 = \frac{\mathbb{E}_t^* \left[R (\ln R)^2 \right]}{\mathbb{E}_t^* \left[R \right]} - \left(\frac{\mathbb{E}_t^* \left[R \ln R \right]}{\mathbb{E}_t^* \left[R \right]} \right)^2.$$

As before, we consider a log utility investor (Martin 2017) and compute risk-neutral expectations via Lemma A1. The objective volatility dynamics in (15) are

ϕ_1	ϕ_2	ϕ_3	\overline{x}	σ_e
0.96	-0.27	0.17	0.98	0.36
(0.05)	(0.07)	(0.05)	(0.14)	(0.01)

The units are non-annualized percentage points. Standard errors are from the simulations below.¹¹

Simulations. We evaluate the model via Monte Carlo simulations. The steps are as follows:

- 1. We simulate the dynamics of volatility via (15). We simulate 100,000 artificial samples of length T = 378 months. In each artificial sample, we use a burn-in period of 100 years.
- 2. We compute forecasts of future volatility via (16), but we replace the objective long-term mean \overline{x} with the misperceived counterpart in (20). In each artificial sample, we initialize the misperceived mean at \overline{x} . When $\theta_F > 0$, these forecasts are distorted by forward rate bias; $\theta_F = 0$ nests rationality.

¹¹We note that any measure of volatility would produce qualitatively similar results: what matters is not the particular measure but rather the mapping to risk premia.

- 3. We map forecasts of future volatility to forecasts of future short rates via (18), but we replace the objective expectation $\mathbb{E}[\cdot]$ with the subjective counterpart $\mathbb{E}^{\theta}[\cdot]$ from the previous step. We estimate the mapping in (18) from a regression of contemporaneous short rates on volatility, as in (17).
- 4. We compute perceptions of current short rates via (21). When $\theta_S \neq 0$, these perceptions are distorted by short rate bias; $\theta_S = 0$ nests rationality.
- 5. We compute current long-term spot rates via (19), but we again replace the objective expectation $\mathbb{E}[\cdot]$ with the subjective counterpart $\mathbb{E}^{\theta}[\cdot]$ from the previous two steps.

With the term structure of spot rates in hand, we can compute forward rates and forecast errors in the usual way.

E.2 Discussion: A Trilemma for Expectation Errors

This appendix continues the discussion in Section 6.3 on how different moments of the data are tied together by the cyclicality of forecast errors.¹² We begin with the Campbell-Shiller price-dividend decomposition in (11). Assume that the expectations $\mathbb{E}_t[\cdot]$ in that decomposition refer to agents subjective beliefs, and $p_t - d_t$ is the observed log price-dividend ratio. Now consider an alternative economy in which all agents have rational expectations. For arbitrary equilibrium variable x_t in the observed data, denote the corresponding variable in the alternative RE economy by x_t^{RE} . Define the wedge between these two variables to be $\tilde{x}_t = x_t - x_t^{RE}$. For example, $p_t - d_t$ is the wedge between the observed price-dividend ratio and the one that would be observed in the alternative economy with RE. Up to a constant, it satisfies

$$\widetilde{p_t - d_t} = \widetilde{CF}_t - \widetilde{F}_t - \widetilde{RF}_t.$$
(A17)

Assume for simplicity that $\widetilde{RF}_t = 0$. The following variance decomposition for the price-dividend wedge therefore holds:

$$\operatorname{var}\left(\widetilde{p_t - d_t}\right) = \operatorname{var}\left(\widetilde{CF}_t\right) + \operatorname{var}\left(\widetilde{\mathcal{F}}_t\right) - 2\operatorname{cov}\left(\widetilde{CF}_t, \widetilde{\mathcal{F}}_t\right).$$
(A18)

Alternatively, one can also use the following decomposition given (A17):

$$\operatorname{var}\left(\widetilde{p_t - d_t}\right) = \operatorname{cov}\left(\widetilde{p_t - d_t}, \widetilde{CF}_t\right) - \operatorname{cov}\left(\widetilde{p_t - d_t}, \widetilde{\mathcal{F}}_t\right).$$
(A19)

The wedges \widetilde{CF}_t and $\widetilde{\mathcal{F}}_t$ can be understood as expectation errors along the lines considered in Section 6: if subjective expectations are too high relative to RE, then the wedge will be positive (and forecast errors, defined as realized – forecast, are likely to be negative). According to either of the decompositions in (A18)–(A19), therefore, one must choose from at most two of the following three features of any model of expectation errors:

- 1. Volatile expectation errors for returns (and/or fundamentals)
- 2. Volatile price-dividend ratio relative to a rational benchmark
- 3. Countercyclical return expectation errors (positive return expectation errors in bad times)

¹²This discussion builds on Campbell (2017), who introduces related trilemmas for present value and portfolio choice.

For example, if excessively positive cash-flow and return forecast revisions occur in good times (after positive news), then $\operatorname{cov}\left(\widetilde{CF}_t, \widetilde{\mathcal{F}}_t\right) > 0$ in (A18). Alternatively, in the version expressed in (A19), positive comovement between price-dividend and forward-rate wedges similarly detracts from a model's ability to generate volatile $\widetilde{p_t} - d_t$. This form of overreaction to *realized outcomes* (cash flows and/or returns) may be intuitively appealing, but it limits a model's ability to speak to variation in the price-dividend ratio through expectation errors alone.¹³

Our empirical results, and our model of expectation errors, instead suggest overreaction of forward rates to fundamentals, rather than realized returns. Unlike realized returns, we find that spot and forward rates *increase* in bad times. The negative covariance between fundamental news and return expectation errors in principle allows for a volatile price-dividend ratio.

¹³For example, Nagel and Xu (2022) obtain a price-dividend ratio volatility about 50% lower than that observed in the data (see their Table 5). Similarly, De la O and Myers (2021) report that in the model of Barberis et al. (2015), "movements in dividend change expectations are almost completely negated by movements in price change expectations. This leads to low variation in the price-dividend difference" (p. 1370); Campbell (2017) provides a related discussion of the Barberis et al. (2015) results.

Appendix Tables and Figures

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Table A1 **Option Sample**

This table reports the country or index name, the abbreviation, the underlying index, the sample period, and the sample length in months for each exchange. The last column indicates whether the exchange is in the main sample. See Appendix B.1 for more details.

Name	Abbrv	Index	Start	End	Length	Main
North America						
United States	USA	S&P 500	199001	202112	384	Υ
Europe						
Belgium	BEL	BEL 20	200201	202109	228	
Switzerland	CHE	SMI	200201	202109	237	Υ
Germany	DEU	DAX	200201	202109	237	Υ
Spain	ESP	IBEX 35	200610	202109	180	Υ
Finland	FIN	OMXH25	200201	202109	237	
France	\mathbf{FRA}	CAC 40	200304	202109	222	Υ
United Kingdom	GBR	FTSE 100	200201	202109	237	Υ
Italy	ITA	FTSE MIB	200610	202109	180	Υ
Netherlands	NLD	AEX	200201	202109	219	
Sweden	SWE	OMXS30	200705	202109	173	
Pan-Europe						
Euro Stoxx 50	SX5E	SX5E	200201	202109	237	Υ
Stoxx Europe 50	SX5P	SX5P	200201	202109	237	
Stoxx Europe 600	SXXP	SXXP	200509	202109	193	
Asia-Pacific						
Australia	AUS	ASX 200	200401	202104	208	Υ
China	CHN	HSCEI	200601	202104	184	
Hong Kong	HKG	Hang Seng	200601	202104	184	Υ
Japan	JPN	Nikkei 225	200405	202104	204	
Korea	KOR	KOSPI 200	200407	202104	202	
Taiwan	TWN	TAIEX	200510	202104	187	

Table A2 Summary Statistics

This table reports summary statistics for option-based (Panel A) and survey-based (Panel B) expectations. The expectations are forward rates and the corresponding realized spot rates for risk premia (on the left) and expected returns (on the right). The option-based horizon is the 6×6 -month forecast error, the Livingston horizon is the 6×6 -month forecast error, and the CFO horizon is the 1×9 -year forecast error. The units are annualized percentage points. The option-based sample is the longest available for each exchange, the Livingston sample is half-yearly from June 1992 to June 2021, and the CFO sample is quarterly from December 2001 to June 2021. See Section 4 for more details.

			PANE	l A. Opt	ION-BASE	D Expect	TATIONS		
	Forwa	rd RP	Spo	t RP	Forwa	ard ER	Spo	t ER	
	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev	Length
North An	nerica								
USA	2.15	0.99	2.17	1.40	5.44	2.32	5.04	2.41	378
Europe									
BEL	2.56	2.22	2.42	2.07	4.13	2.79	3.59	2.82	213
CHE	1.88	0.80	1.96	1.34	2.60	1.50	2.26	1.85	231
DEU	2.63	1.07	2.85	1.86	4.27	2.12	4.10	2.74	231
ESP	2.89	1.17	3.37	1.83	4.19	2.19	4.19	2.73	168
FIN	2.38	1.85	2.79	2.21	4.02	2.65	4.03	2.85	231
\mathbf{FRA}	2.44	1.04	2.57	1.56	3.97	2.07	3.71	2.42	215
GBR	2.12	1.00	2.21	1.51	4.76	2.13	4.33	2.59	224
ITA	3.17	1.35	3.60	1.67	4.42	2.23	4.39	2.40	174
NLD	2.43	1.26	2.85	2.18	3.95	2.40	3.93	3.20	202
SWE	2.44	1.41	2.79	1.80	3.77	2.68	3.79	2.83	160
Pan-Euro	pe								
SX5E	2.65	1.08	2.89	1.83	4.29	2.10	4.14	2.69	231
SX5P	2.11	1.48	2.23	1.62	3.75	2.51	3.47	2.64	231
SXXP	2.05	1.53	2.14	1.63	3.44	2.57	3.12	2.65	185
Asia-Pacif	ìc								
AUS	1.84	1.20	1.97	1.39	5.95	2.39	5.65	2.65	197
CHN	3.90	3.29	4.24	4.01	5.92	3.64	5.61	4.42	178
HKG	2.85	1.93	2.98	2.56	4.86	2.26	4.35	2.98	178
JPN	2.60	1.57	2.93	2.20	3.15	1.76	3.21	2.35	197
KOR	2.14	1.85	2.23	2.16	5.40	2.95	5.13	3.02	194
TWN	2.36	1.45	2.39	1.90	3.84	1.70	3.37	2.11	181
			PANE	l B. Surv	VEY-BASE	D EXPECT	TATIONS		
	Forwa	rd RP	Spo	t RP	Forwa	ard ER	Spo	t ER	
	Mean	StDev	Mean	StDev	Mean	StDev	Mean	StDev	Length
Livingstor	n Survey	,							
Option	2.16	1.01	2.08	1.20	5.20	2.16	4.79	2.24	59
Survey	3.15	4.96	3.52	5.35	5.73	3.81	5.84	4.14	59
CFO Surv	ey								
Option	2.29	1.04	2.31	1.56	4.12	1.42	3.94	1.92	74
Survey	3.35	1.08	3.54	1.15	6.83	0.82	6.67	0.83	74

Table A3 **Alternative Integration Bounds**

This table reports estimates with alternative integration bounds for option-based risk premia (Panel A) and expected returns (Panel B). Mincer-Zarnowitz regressions test $H_0: \beta_1 = 1$, as in Table 2. The average forecast error tests $H_0: \bar{\varepsilon}_t = 0$, as in Table 4. Error-predictability regressions test $H_0: \beta_1 = 0$, as in Table 5. The horizon is the 6-month spot rate, 6 months from now. The units are annualized percentage points. All regressions include exchange fixed effects and compute a within R^2 . Standard errors are clustered by exchange and date. The sample is the longest available for each exchange in the main sample. See Appendix C.1 for more details.

		Panel A. Risk Premia $\tilde{\mu}_{t+6}^{(6)}$											
		Mincer-Z	arnowitz		Ave	erage Erro	or	Error Predictability					
	β_1	$se(\beta_1)$	p-val	R^2	$\tilde{\varepsilon}_t$	$se(\tilde{\varepsilon}_t)$	<i>p</i> -val	β_1	$se(\beta_1)$	p-val	R^2	Ν	
Without Extrapolation													
Truncation: $0.75 \leq K/P \leq 1.25$	0.92	0.12		0.19	0.44	0.088	***	0.066	0.040		0.01	2140	
Truncation: $0.65 \leq K/P \leq 1.35$	0.78	0.091	**	0.18	0.34	0.094	***	-0.019	0.039		0.00	2139	
Truncation: $0.55 \leq K/P \leq 1.45$	0.68	0.074	***	0.17	0.26	0.097	**	-0.082	0.041	*	0.01	2137	
Truncation: $0.45 \leq K/P \leq 1.55$	0.63	0.065	***	0.16	0.22	0.099	*	-0.12	0.042	**	0.02	2140	
Truncation: $0.35 \leq K/P \leq 1.65$	0.59	0.059	***	0.15	0.20	0.10	*	-0.15	0.044	***	0.03	2138	
Observable Strike Prices	0.56	0.053	***	0.15	0.19	0.10	*	-0.17	0.043	***	0.04	2140	
With Extrapolation													
Static Integration Bounds	0.56	0.056	***	0.15	0.14	0.10		-0.15	0.041	***	0.04	2244	
Dynamic Integration Bounds	0.56	0.055	***	0.15	0.16	0.11		-0.17	0.046	***	0.03	2241	
Baseline: $0.10 \le K/P \le 2.00$	0.56	0.055	***	0.15	0.17	0.11		-0.16	0.047	***	0.03	2227	

		Panel B. Expected Returns $\mu_{t+6}^{(6)}$											
		Mincer-Za	arnowitz		Ave	erage Erro	or	Error Predictability					
	β_1	$se(\beta_1)$	p-val	\mathbb{R}^2	ε_t	$se(\varepsilon_t)$	<i>p</i> -val	β_1	$se(\beta_1)$	p-val	R^2	Ν	
Without Extrapolation													
Truncation: $0.75 \leq K/P \leq 1.25$	1.04	0.052		0.77	-0.0079	0.088		-0.12	0.048	**	0.02	2140	
Truncation: $0.65 \leq K/P \leq 1.35$	1.02	0.053		0.71	-0.12	0.094		-0.18	0.043	***	0.04	2139	
Truncation: $0.55 \leq K/P \leq 1.45$	0.99	0.054		0.67	-0.19	0.097	*	-0.23	0.044	***	0.07	2137	
Truncation: $0.45 \leq K/P \leq 1.55$	0.96	0.055		0.64	-0.23	0.099	**	-0.26	0.045	***	0.09	2140	
Truncation: $0.35 \leq K/P \leq 1.65$	0.94	0.056		0.63	-0.25	0.10	**	-0.29	0.046	***	0.10	2138	
Observable Strike Prices	0.93	0.058		0.61	-0.26	0.10	**	-0.31	0.046	***	0.12	2140	
With Extrapolation													
Static Integration Bounds	0.92	0.060		0.60	-0.31	0.100	**	-0.26	0.039	***	0.12	2244	
Dynamic Integration Bounds	0.91	0.061		0.59	-0.29	0.10	**	-0.30	0.047	***	0.11	2241	
Baseline: $0.10 \le K/P \le 2.00$	0.91	0.061		0.59	-0.28	0.10	**	-0.29	0.047	***	0.10	2227	

Table A4Alternative Measures

This table reports estimates with alternative measures for option-based risk premia (Panel A) and expected returns (Panel B). Mincer-Zarnowitz regressions test $H_0: \beta_1 = 1$, as in Table 2. The average forecast error tests $H_0: \bar{\varepsilon}_t = 0$, as in Table 4. Error-predictability regressions test $H_0: \beta_1 = 0$, as in Table 5. The horizon is the 6-month spot rate, 6 months from now. The units are annualized percentage points. Panel regressions, in the main sample, include exchange fixed effects, compute a within R^2 , and report standard errors clustered by exchange and date. This sample is the longest available for each exchange. Time-series regressions, in the U.S. sample, report Newey-West standard errors with $L = [1.3 \times T^{1/2}]$ lags and fixed-*b p*-values, following Lazarus et al. (2018). This sample is from January 1990 to June 2021. See Appendix C.2 for more details.

	Panel A. Risk Premia $\tilde{\mu}_{t+6}^{(6)}$											
		Mincer-Z	arnowit	Z	Ave	erage Eri	or	E	rror Pred	ictabilit	y	
	β_1	$se(\beta_1)$	p-val	R^2	$\tilde{\varepsilon}_t$	$se(\tilde{\varepsilon}_t)$	<i>p</i> -val	β_1	$se(\beta_1)$	p-val	R^2	Ν
Main Sample												
Baseline Filters/Surface	0.56	0.055	***	0.15	0.17	0.11		-0.16	0.047	***	0.03	2227
Outlier Filter	0.60	0.055	***	0.17	0.18	0.098		-0.20	0.050	***	0.05	2241
Open Interest Filter: After 199601	0.52	0.057	***	0.14	0.16	0.11		-0.18	0.045	***	0.04	2033
Outlier/Open Interest Filter: After 199601	0.56	0.051	***	0.15	0.19	0.097	*	-0.21	0.050	***	0.06	2040
Volatility Surface: After 199601	0.57	0.051	***	0.17	0.087	0.095		-0.21	0.053	***	0.06	2163
SVI Surface: U.S. and SX5E	0.59	0.047	*	0.17	0.051	0.11		-0.19	0.039		0.04	609
U.S. Sample												
Bid-Ask Midpoint	0.67	0.096	***	0.22	0.021	0.15		-0.17	0.067	**	0.03	378
Bid Prices	0.64	0.099	***	0.21	0.034	0.14		-0.15	0.078	*	0.03	378
Ask Prices	0.66	0.089	***	0.23	0.0051	0.15		-0.18	0.060	***	0.04	378
									(6)			

	Panel B. Expected Returns $\mu_{t+6}^{(6)}$											
]	Mincer-Zarnowitz				rage Err	or	Eı	rror Pred	ictabilit	У	
	$\beta_1 se(\beta_1) p$ -val R^2				ε_t	$se(\varepsilon_t)$	p-val	β_1	$se(\beta_1)$	p-val	R^2	Ν
Main Sample												
Baseline Filters/Surface	0.91	0.061		0.59	-0.28	0.10	**	-0.29	0.047	***	0.10	2227
Outlier Filter	0.92	0.062		0.62	-0.27	0.098	**	-0.34	0.050	***	0.15	2241
Open Interest Filter: After 199601	0.91	0.062		0.59	-0.26	0.10	**	-0.31	0.043	***	0.12	2033
Outlier/Open Interest Filter: After 199601	0.92	0.064		0.62	-0.24	0.096	**	-0.35	0.048	***	0.16	2040
Volatility Surface: After 199601	0.93	0.065		0.63	-0.34	0.098	***	-0.36	0.052	***	0.16	2163
SVI Surface: U.S. and SX5E	0.89	0.048		0.65	-0.36	0.11		-0.28	0.049		0.10	609
U.S. Sample												
Bid-Ask Midpoint	0.88	0.073		0.71	-0.40	0.17	**	-0.20	0.091	**	0.04	378
Bid Prices	0.87	0.070	*	0.73	-0.38	0.17	**	-0.18	0.11		0.04	378
Ask Prices	0.87	0.076		0.69	-0.41	0.18	**	-0.21	0.077	**	0.05	378

Table A5Split-Sample Regressions

This table reports estimates in split samples for option-based risk premia (Panel A) and expected returns (Panel B). Mincer-Zarnowitz regressions test $H_0: \beta_1 = 1$, as in Table 2. The average forecast error tests $H_0: \bar{\varepsilon}_t = 0$, as in Table 4. Error-predictability regressions test $H_0: \beta_1 = 0$, as in Table 5. The horizon is the 6-month spot rate, 6 months from now. The units are annualized percentage points. Each regression reports Newey-West standard errors with $L = \lfloor 1.3 \times T^{1/2} \rfloor$ lags and fixed-b p-values, following Lazarus et al. (2018). See Appendix C.3 for more details.

					Pane	l A. Risi	k Premi	A $\tilde{\mu}_{t+6}^{(6)}$				
		Mincer-Za	arnowitz		Av	erage Eri	or	Error Predictability				
	β_1	$se(\beta_1)$	p-val	R^2	$\tilde{\varepsilon}_t$	$se(\tilde{\varepsilon}_t)$	p-val	β_1	$se(\beta_1)$	p-val	R^2	Ν
U.S. Sample												
Baseline: 199001 to 202106	0.67	0.096	***	0.22	0.021	0.15		-0.17	0.067	**	0.03	378
First Half: 199001 to 200509	0.82	0.12		0.38	0.025	0.15		-0.055	0.074		-0.00	189
Second Half: 200510 to 202106	0.56	0.11	***	0.14	0.017	0.24		-0.22	0.075	**	0.05	189
Second Half: 200907 to 201907	0.50	0.10	***	0.20	-0.40	0.12	***	-0.37	0.089	***	0.16	121
]	Panel B	. Expect	fed Ret	URNS $\mu_{t+}^{(6)}$) 6			
		Mincer-Za	arnowitz		Av	erage Err	ror	F	Error Pred	ictability	r	
	β_1	$se(\beta_1)$	p-val	R^2	ε_t	$se(\varepsilon_t)$	<i>p</i> -val	β_1	$se(\beta_1)$	p-val	R^2	Ν
U.S. Sample												
Baseline: 199001 to 202106	0.88	0.073		0.71	-0.40	0.17	**	-0.20	0.091	**	0.04	378
First Half: 199001 to 200509	0.85	0.081		0.72	-0.58	0.25	**	0.023	0.12		-0.00	189
Second Half: 200510 to 202106	0.92	0.14		0.48	-0.22	0.18		-0.32	0.079	***	0.13	189
Second Half: 200907 to 201907	0.58	0.20	*	0.23	-0.50	0.13	***	-0.47	0.074	***	0.23	121

Table A6Short-Horizon Regressions

This table reports estimates at short horizons for option-based risk premia (Panel A) and expected returns (Panel B). Mincer-Zarnowitz regressions test $H_0: \beta_1 = 1$, as in Table 2. The average forecast error tests $H_0: \bar{\varepsilon}_t = 0$, as in Table 4. Error-predictability regressions test $H_0: \beta_1 = 0$, as in Table 5. The horizon is the *m*-month spot rate, *n* months from now. The units are annualized percentage points. Panel regressions, in the main sample, include exchange fixed effects, compute a within R^2 , and report standard errors clustered by exchange and date. This sample is the longest available for each exchange. Time-series regressions, in the U.S. sample, report Newey-West standard errors with $L = \lfloor 1.3 \times T^{1/2} \rfloor$ lags and fixed-*b p*-values, following Lazarus et al. (2018). This sample is from January 1990 to June 2021. See Appendix C.3 for more details.

		Panel A. Risk Premia $\tilde{\mu}_{t+n}^{(m)}$											
			Mincer-Za	arnowitz		Av	verage Erro	or		Error Pred	ictability		
		β_1	$se(\beta_1)$	p-val	R^2	$\tilde{arepsilon}_t$	$se(\tilde{\varepsilon}_t)$	<i>p</i> -val	β_1	$se(\beta_1)$	p-val	R^2	Ν
Main Sa	ample												
n = 1	m = 1	0.86	0.062	**	0.48	0.068	0.10		-0.17	0.049	***	0.03	2268
n = 1	m=2	0.86	0.055	**	0.53	0.018	0.087		-0.17	0.048	***	0.04	2269
n=2	m = 1	0.71	0.067	***	0.27	0.021	0.13		-0.29	0.067	***	0.06	2269
U.S. Sa	mple												
n = 1	m = 1	0.88	0.067		0.52	-0.020	0.066		-0.14	0.075	*	0.02	381
n = 1	m = 2	0.89	0.050	**	0.60	-0.071	0.051		-0.12	0.052	**	0.02	382
n=2	m = 1	0.73	0.085	***	0.30	-0.14	0.10		-0.27	0.085	***	0.05	382
						Panel B	. Expect	ed Retui	RNS $\mu_{t+n}^{(m)}$				
			Mincer-Za	$\operatorname{arnowitz}$		Average Error				Error Predictability			
		β_1	$se(\beta_1)$	<i>p</i> -val	R^2	ε_t	$se(\varepsilon_t)$	<i>p</i> -val	β_1	$se(\beta_1)$	<i>p</i> -val	R^2	Ν
Main Sa	ample												
n = 1	m = 1	0.94	0.059		0.67	-0.070	0.11		-0.24	0.051	***	0.05	2268
n = 1	m = 2	0.93	0.049		0.73	-0.14	0.086		-0.23	0.048	***	0.07	2269
n=2	m = 1	0.87	0.065	*	0.54	-0.28	0.13	*	-0.40	0.070	***	0.11	2269
U.S. Sa	mple												
n = 1	m = 1	0.98	0.024		0.78	-0.15	0.064	**	-0.20	0.058	***	0.04	381
n = 1	m = 2	0.97	0.016		0.84	-0.20	0.045	***	-0.15	0.042	***	0.04	382
n=2	m = 1	0.94	0.032	*	0.68	-0.40	0.089	***	-0.32	0.065	***	0.08	382

Table A7Alternative Horizons

This table reports estimates at alternative horizons for option-based risk premia (Panel A) and expected returns (Panel B). Mincer-Zarnowitz regressions test $H_0: \beta_1 = 1$, as in Table 2. The average forecast error tests $H_0: \bar{\varepsilon}_t = 0$, as in Table 4. Error-predictability regressions test $H_0: \beta_1 = 0$, as in Table 5. The horizon is the *m*-month spot rate, *n* months from now. The units are annualized percentage points. All regressions include exchange fixed effects and compute a within R^2 . Standard errors are clustered by exchange and date. The sample is the longest available for each exchange in the main sample. See Appendix C.3 for more details.

		Panel A. Risk Premia $\tilde{\mu}_{t+n}^{(m)}$												
		Mincer-Z	arnowitz		A	verage Erro	or	-	Error Predi	ctability				
	β_1	$se(\beta_1)$	p-val	\mathbb{R}^2	$\tilde{\varepsilon}_t$	$se(\tilde{\varepsilon}_t)$	<i>p</i> -val	β_1	$se(\beta_1)$	p-val	R^2	Ν		
4-Month Horiz	on: $n+m$	a = 4												
n=1 $m=3$	0.90	0.048	*	0.62	0.044	0.073		-0.12	0.040	**	0.03	2269		
n=2 $m=2$	0.77	0.058	***	0.35	0.055	0.11		-0.20	0.053	***	0.04	2268		
n=3 $m=1$	0.68	0.065	***	0.20	0.094	0.14		-0.24	0.055	***	0.03	2235		
5-Month Horiz	on: $n+m$	t = 5												
n=1 $m=4$	0.94	0.043		0.67	0.067	0.064		-0.075	0.033	*	0.01	2269		
n = 2 $m = 3$	0.83	0.053	**	0.42	0.12	0.10		-0.13	0.043	**	0.02	2268		
n = 3 $m = 2$	0.74	0.063	***	0.25	0.16	0.13		-0.15	0.049	**	0.02	2241		
n=4 $m=1$	0.62	0.078	***	0.13	0.27	0.15		-0.19	0.053	***	0.02	2236		
6-Month Horiz	on: $n+m$	t = 6												
n = 1 $m = 5$	0.95	0.037		0.69	0.067	0.058		-0.054	0.029	*	0.01	2269		
n = 2 $m = 4$	0.86	0.047	**	0.46	0.14	0.091		-0.094	0.036	**	0.01	2268		
n = 3 $m = 3$	0.78	0.058	***	0.30	0.21	0.12		-0.11	0.044	**	0.01	2249		
n = 4 $m = 2$	0.67	0.072	***	0.17	0.27	0.14	*	-0.15	0.050	**	0.02	2238		
n = 5 $m = 1$	0.57	0.076	***	0.09	0.32	0.17	*	-0.20	0.057	***	0.02	2228		
9-Month Horiz	on: $n+m$	t = 9												
n = 3 $m = 6$	0.81	0.055	***	0.38	0.15	0.087		-0.065	0.036		0.01	2257		
n = 4 $m = 5$	0.73	0.066	***	0.26	0.21	0.10	*	-0.10	0.044	**	0.01	2242		
n = 5 $m = 4$	0.65	0.071	***	0.17	0.26	0.12	*	-0.14	0.049	**	0.02	2232		
n=6 $m=3$	0.55	0.071	***	0.10	0.28	0.14	*	-0.17	0.051	***	0.02	2224		
12-Month Hori	zon: $n + n$	m = 12												
n = 3 $m = 9$	0.81	0.049	***	0.44	0.094	0.070		-0.059	0.036		0.01	2258		
n = 6 $m = 6$	0.56	0.055	***	0.15	0.17	0.11		-0.16	0.047	***	0.03	2227		
n=9 $m=3$	0.39	0.097	***	0.05	0.25	0.15		-0.24	0.052	***	0.04	2191		

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					Panel B. Expected Returns $\mu_{t+n}^{(m)}$							
		Mincer-Za	arnowitz		Av	verage Erro	or		Error Pred	ictability		
	β_1	$se(\beta_1)$	<i>p</i> -val	R^2	ε_t	$se(\varepsilon_t)$	<i>p</i> -val	β_1	$se(\beta_1)$	<i>p</i> -val	R^2	Ν
4-Month Horizon	: n+m	4 = 4										
n = 1 $m = 3$	0.96	0.042		0.80	-0.074	0.073		-0.17	0.039	***	0.06	2269
n=2 $m=2$	0.90	0.057		0.62	-0.21	0.11	*	-0.30	0.056	***	0.09	2268
n=3 $m=1$	0.89	0.068		0.50	-0.26	0.14	*	-0.36	0.059	***	0.08	2235
5-Month Horizon	n+m	z = 5										
n=1 $m=4$	0.99	0.038		0.84	-0.027	0.063		-0.12	0.033	***	0.04	2269
n = 2 $m = 3$	0.95	0.052		0.69	-0.080	0.097		-0.22	0.046	***	0.06	2268
n = 3 $m = 2$	0.94	0.063		0.57	-0.13	0.12		-0.27	0.053	***	0.06	2241
n=4 $m=1$	0.92	0.080		0.45	-0.11	0.14		-0.33	0.057	***	0.06	2236
6-Month Horizon	n+m	b = 6										
n = 1 $m = 5$	0.99	0.035		0.86	-0.027	0.057		-0.10	0.029	***	0.03	2269
n=2 $m=4$	0.97	0.049		0.73	-0.039	0.086		-0.18	0.040	***	0.05	2268
n=3 $m=3$	0.97	0.057		0.63	-0.043	0.11		-0.22	0.048	***	0.05	2249
n = 4 $m = 2$	0.95	0.071		0.52	-0.082	0.13		-0.29	0.052	***	0.06	2238
n = 5 $m = 1$	0.93	0.082		0.42	-0.15	0.15		-0.36	0.059	***	0.07	2228
9-Month Horizon	n+m	b = 9										
n=3 $m=6$	0.97	0.046		0.73	-0.075	0.082		-0.16	0.036	***	0.05	2257
n = 4 $m = 5$	0.97	0.055		0.66	-0.082	0.096		-0.22	0.040	***	0.06	2242
n = 5 $m = 4$	0.97	0.063		0.59	-0.10	0.11		-0.27	0.046	***	0.07	2232
n = 6 $m = 3$	0.94	0.071		0.51	-0.17	0.13		-0.31	0.053	***	0.08	2224
12-Month Horizo	n: $n + r$	m = 12										
n=3 $m=9$	0.96	0.039		0.78	-0.14	0.067	*	-0.15	0.034	***	0.06	2258
n=6 $m=6$	0.91	0.061		0.59	-0.28	0.10	**	-0.29	0.047	***	0.10	2227
n=9 $m=3$	0.81	0.077	**	0.41	-0.46	0.15	**	-0.41	0.062	***	0.12	2191

Table A7Alternative Horizons (Continued)

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Table A8Alternative Samples

This table reports estimates in alternative samples for option-based risk premia (Panel A) and expected returns (Panel B). Mincer-Zarnowitz regressions test $H_0: \beta_1 = 1$, as in Table 2. The average forecast error tests $H_0: \bar{\varepsilon}_t = 0$, as in Table 4. Error-predictability regressions test $H_0: \beta_1 = 0$, as in Table 5. The horizon is the 1-month spot rate, 2 months from now. The units are annualized percentage points. All regressions include exchange fixed effects and compute a within R^2 . Standard errors are clustered by exchange and date. The sample is the longest available for each exchange. See Appendix C.3 for more details.

	Panel A. Risk Premia $\tilde{\mu}_{t+6}^{(6)}$											
	Mincer-Zarnowitz				Av	Average Error			Error Predictability			
	β_1	$se(\beta_1)$	p-val	R^2	$\tilde{\varepsilon}_t$	$se(\tilde{\varepsilon}_t)$	<i>p</i> -val	β_1	$se(\beta_1)$	p-val	R^2	Ν
Baseline												
Main: 10 Exchanges	0.76	0.059	***	0.30	0.047	0.13		-0.24	0.059	***	0.04	2198
Full: 20 Exchanges	0.76	0.050	***	0.30	0.14	0.13		-0.24	0.050	***	0.04	4033
Eurozone												
Main: 5 Exchanges	0.73	0.076	**	0.28	0.070	0.14		-0.27	0.076	**	0.05	1040
Full: 8 Exchanges	0.72	0.062	***	0.29	0.12	0.15		-0.28	0.062	***	0.06	1568
Europe												
Main: 7 Exchanges	0.73	0.067	***	0.28	0.057	0.13		-0.27	0.067	***	0.05	1510
Full: 13 Exchanges	0.72	0.054	***	0.29	0.084	0.13		-0.28	0.054	***	0.06	2619
Asia-Pacific												
Main: 2 Exchanges	0.88	0.12		0.36	0.21	0.15		-0.12	0.12		0.01	319
Full: 6 Exchanges	0.86	0.078		0.34	0.36	0.17	*	-0.14	0.078		0.01	1045
Excludes U.S.												
Main: 9 Exchanges	0.76	0.063	***	0.30	0.083	0.13		-0.24	0.063	***	0.04	1829
Full: 19 Exchanges	0.76	0.051	***	0.30	0.16	0.14		-0.24	0.051	***	0.04	3664

	Panel B. Expected Returns $\mu_{t+6}^{(6)}$											
	Mincer-Zarnowitz				Av	Average Error			Error Predictability			
	β_1	$se(\beta_1)$	p-val	\mathbb{R}^2	ε_t	$se(\varepsilon_t)$	<i>p</i> -val	β_1	$se(\beta_1)$	p-val	\mathbb{R}^2	Ν
Baseline												
Main: 10 Exchanges	0.90	0.063		0.57	-0.26	0.12	*	-0.36	0.064	***	0.09	2198
Full: 20 Exchanges	0.89	0.066		0.55	-0.17	0.13		-0.35	0.057	***	0.09	4033
Eurozone												
Main: 5 Exchanges	0.87	0.080		0.53	-0.24	0.14		-0.40	0.085	***	0.11	1040
Full: 8 Exchanges	0.85	0.078		0.53	-0.20	0.15		-0.41	0.071	***	0.12	1568
Europe												
Main: 7 Exchanges	0.87	0.076		0.54	-0.24	0.13		-0.41	0.077	***	0.11	1510
Full: 13 Exchanges	0.86	0.074	*	0.56	-0.23	0.13		-0.42	0.064	***	0.13	2619
Asia-Pacific												
Main: 2 Exchanges	0.96	0.071		0.59	-0.21	0.16		-0.23	0.10		0.03	319
Full: 6 Exchanges	0.95	0.081		0.50	0.036	0.17		-0.21	0.076	**	0.03	1045
Excludes U.S.												
Main: 9 Exchanges	0.89	0.071		0.55	-0.23	0.13		-0.37	0.069	***	0.09	1829
Full: 19 Exchanges	0.88	0.071		0.54	-0.15	0.14		-0.36	0.059	***	0.09	3664

Table A9Power Utility Regressions

This table reports estimates from the standpoint of an unconstrained power utility investor fully invested in the market for option-based risk premia (Panel A) and expected returns (Panel B). Mincer-Zarnowitz regressions test $H_0: \beta_1 = 1$, as in Table 2. The average forecast error tests $H_0: \bar{\varepsilon}_t = 0$, as in Table 4. Error-predictability regressions test $H_0: \beta_1 = 0$, as in Table 5. The horizon is the 6-month spot rate, 6 months from now. The units are annualized percentage points. Panel regressions, in the main sample, include exchange fixed effects, compute a within R^2 , and report standard errors clustered by exchange and date. This sample is the longest available for each exchange. Time-series regressions, in the U.S. sample, report Newey-West standard errors with $L = \left[1.3 \times T^{1/2}\right]$ lags and fixed-*b p*-values, following Lazarus et al. (2018). This sample is from January 1990 to June 2021. See Appendix D.1 for more details.

					Pane	el A. Ris	K Prem	IA $ ilde{\mu}_{t+6}^{(6)}$				
		Mincer-Zarnowitz				erage Err	or	E	rror Predi	ctability		
	β_1	$se(\beta_1)$	p-val	R^2	$\tilde{\varepsilon}_t$	$se(\tilde{\varepsilon}_t)$	<i>p</i> -val	β_1	$se(\beta_1)$	p-val	R^2	Ν
Main Sam	ple											
$\gamma = 0.75$	0.51	0.059	***	0.13	0.045	0.057		-0.19	0.045	***	0.04	2224
$\gamma = 1.00$	0.56	0.055	***	0.15	0.17	0.11		-0.16	0.046	***	0.03	2220
$\gamma = 1.25$	0.60	0.057	***	0.15	0.36	0.15	**	-0.13	0.050	**	0.02	2215
$\gamma = 1.50$	0.62	0.062	***	0.16	0.59	0.20	**	-0.10	0.054	*	0.01	2206
$\gamma = 2.00$	0.64	0.075	***	0.15	1.14	0.29	***	-0.044	0.057		0.00	2181
U.S. Samp	le											
$\gamma = 0.75$	0.61	0.095	***	0.21	-0.022	0.083		-0.20	0.070	***	0.05	378
$\gamma = 1.00$	0.67	0.096	***	0.22	0.021	0.15		-0.17	0.067	**	0.03	378
$\gamma = 1.25$	0.72	0.099	**	0.23	0.12	0.20		-0.13	0.064	*	0.02	378
$\gamma = 1.50$	0.76	0.10	**	0.24	0.25	0.26		-0.099	0.063		0.01	378
$\gamma = 2.00$	0.85	0.10		0.26	0.57	0.35		-0.048	0.060		0.00	378
$\gamma = 2.50$	0.91	0.10		0.27	0.94	0.44	*	-0.0087	0.057		-0.00	378
$\gamma = 3.00$	0.97	0.10		0.28	1.32	0.52	**	0.022	0.056		-0.00	378
					Panel B	B. Expec	ted Rei	TURNS $\mu_{t+1}^{(6)}$	6			
		Mincer-Z	arnowitz		Av	Average Error Erro			rror Predi	ror Predictability		
	β_1	$se(\beta_1)$	p-val	R^2	ε_t	$se(\varepsilon_t)$	<i>p</i> -val	β_1	$se(\beta_1)$	p-val	R^2	Ν
Main Sam	ple											
$\gamma = 0.75$	0.93	0.042		0.75	-0.40	0.061	***	-0.42	0.056	***	0.17	2224
$\gamma = 1.00$	0.91	0.061		0.59	-0.28	0.10	**	-0.30	0.046	***	0.11	2220
$\gamma = 1.25$	0.90	0.075		0.49	-0.084	0.15		-0.22	0.048	***	0.07	2215
$\gamma = 1.50$	0.90	0.088		0.43	0.15	0.19		-0.17	0.052	***	0.04	2206
$\gamma = 2.00$	0.88	0.11		0.36	0.69	0.28	**	-0.093	0.057		0.01	2181
U.S. Samp	le											
$\gamma = 0.75$	0.89	0.061	*	0.84	-0.44	0.14	***	-0.24	0.13	*	0.04	378
$\dot{\gamma} = 1.00$	0.88	0.073		0.71	-0.40	0.17	**	-0.20	0.091	**	0.04	378
$\gamma = 1.25$	0.87	0.083		0.61	-0.30	0.21		-0.15	0.077	*	0.03	378
$\gamma = 1.50$	0.87	0.090		0.53	-0.17	0.26		-0.12	0.071		0.02	378
$\gamma = 2.00$	0.90	0.096		0.45	0.15	0.34		-0.066	0.063		0.00	378
$\gamma = 2.50$	0.93	0.098		0.41	0.52	0.42		-0.024	0.059		-0.00	378
$\gamma = 3.00$	0.96	0.098		0.39	0.90	0.50	*	0.0077	0.056		-0.00	378

Table A10 Long-Horizon Forecast Errors

This table reports estimates of forecast error decay for option-based risk premia. The time-t decay $\phi_t^{(n,m)}$ is the ratio of long to short-horizon forecast errors:

$$\mathbb{E}_t \left[\tilde{\varepsilon}_{t+n}^{(m)} \right] = \phi_t^{(n,m)} \, \mathbb{E}_t \left[\tilde{\varepsilon}_{t+12}^{(12)} \right].$$

The table below reports the median decay $\phi^{(n,m)}$ by horizon:

$$\phi^{(n,m)} = median\left\{ \left| \phi_t^{(n,m)} \right| \right\}.$$

The predicted forecast error $\mathbb{E}_t[\cdot]$ is from a time-series regression of future realized forecast errors on current forward rates: the predictor is the 6 × 6-month forward rate for n = 12 and the $n - 12 \times 12$ -month forward rate for $n \ge 24$. The sample begins in September 2005 and is the longest available for each horizon in the Eurozone (Euro Stoxx 50). See Appendix D.2 for more details.

	<i>n</i> -months										
m-months	12	24	36	48	60	72	84				
12	1.00	1.84	1.52	0.70	1.28	1.74	1.59				
24	1.00	1.81	1.55	0.85	1.26	1.94					
36	1.00	1.85	1.63	0.83	1.64		I				
48	1.00	1.98	1.76	1.35							
60	1.00	1.97	1.93								
72	1.00	2.03		•							
84	1.00										

Figure A1 Truncation Bias in Theory

This figure reports truncation bias in the Black-Scholes (left panel) and SVJ (right panel) models. Panel A truncates the implied volatility surface at the specified bounds (truncation bounds). Panel B extrapolates the implied volatility surface from the specified bounds (extrapolation bounds). The x-axis units are simple moneyness K/P units from the index price. The y-axis units are non-annualized basis points. The bar units are volatility standard deviations from the forward price: $\frac{K/F-1}{\sigma\sqrt{\tau}}$. Black-Scholes parameters: P = 100, r = 0.05, q = 0.00. SVJ parameters under the risk-neutral measure are from Bakshi, Cao, and Chen (1997):

$ heta_v$	κ_v	σ_v	ρ	μ_J	σ_J	λ
0.040	2.030	0.380	-0.570	-0.050	0.070	0.590

Good times correspond to low volatility (IV = 0.20 or $\sqrt{v} = 0.20$), bad times to high volatility (IV = 0.60 or $\sqrt{v} = 0.60$). See Appendix C.1 for more details.



PANEL B. EXTRAPOLATION OF IMPLIED VOLATILITY SURFACE



Figure A2 Minimum/Maximum Strike Price: U.S. Sample

This figure plots the minimum/maximum strike price by maturity bin in the U.S. sample. Each bar is the monthly median from daily data. This sample is longest available for each maturity. The black line is the unconditional median from daily data. This sample is the longest available for all maturities. The minimum is the 1st percentile from out-of-the-money put options. The maximum is the 99th percentile from out-of-the-money call options. The units are risk-neutral standard deviations from the forward price: $\frac{K/F-1}{\sigma\sqrt{\tau}}$. See Appendix C.1 for more details.



Figure A3 Alternative Integration Bounds

This figure plots 6×6 -month forward rates $\tilde{f}_t^{(6,6)}$ (top panel) and the corresponding realized 6-month spot rates $\tilde{\mu}_{t+6}^{(6)}$ (bottom panel) with alternative integration bounds. Spot and forward rates are for option-based risk premia. Gray bands are NBER recessions. The sample is from January 1990 to June 2021 in the U.S. See Appendix C.1 for more details.



Figure A4 Cyclical Variation in Survey-Based Risk Premia

This figure plots 1-year spot rates (red) and 1×9 -year forward rates (blue) for CFO (top panel) and Vanguard (bottom panel) survey expectations. Survey expectations of Vanguard retail investors are from Giglio et al. (2021). Spot and forward rates are for risk premia. Gray bands are NBER recessions. The CFO sample is quarterly from December 2001 to March 2022. The Vanguard sample is bimonthly from February 2017 to December 2019 and monthly from February 2020 to April 2020. See Section 4.2 for more details.



- 1-Year Spot Risk Premium - 1 \times 9-Year Forward Risk Premium

Figure A5 Cyclical Variation in Forward Rates

This figure reports regressions of contemporaneous forward rates on state variables for risk premia (left panel) and expected returns (right panel). The bar height reports the slope *t*-stat. The bar label reports the corresponding *p*-value for H_0 : $\beta_1 = 0$. On the left-hand side, the option-based horizon is the 6 × 6-month forward rate, the Livingston horizon is the 6 × 6-month forward rate, and the CFO horizon is the 1 × 9-year forward rate. On the right-hand side, 1/CAPE is the cyclically-adjusted earnings yield (obtained from Robert Shiller's website), VIX² is the squared 1-year CBOE Volatility Index (obtained from the CBOE's website), and NBER is a recession indicator. Each regression reports Newey-West standard errors with $L = [1.3 \times T^{1/2}]$ lags and fixed-*b p*-values, following Lazarus et al. (2018). The option-based sample is monthly from January 1990 to June 2021, the Livingston sample is half-yearly from June 1992 to June 2021, and the CFO sample is quarterly from December 2001 to March 2022. See Section 4.3 for more details.



Figure A6 Model Calibration: Regression Fit

This figure reports estimates in the calibrated model of expectation errors. The model is calibrated with option-based risk premia in the U.S. sample. The black line is the simulated population R^2 in a single long sample with T = 37,800,000 months. The gray ribbon is the simulated 95% confidence interval in 100,000 short samples with T = 378 months. The red circle is the simulated R^2 under rational expectations with $\theta_F = 0$. The green square is the simulated R^2 under forward rate bias with $\theta_F = 0.0505$. The teal triangle is the observed R^2 in the data with option-based expectations. The purple diamond is the observed R^2 in the data with control expectations. The horizon is the 6-month spot rate, 6 months from now. The units are annualized percentage points. Short rate bias $\theta_S = 0$ for all simulated R^2 s. See Appendix E.1 for more details.



- Model Simulation
- Rational Expectations: $\theta_F = 0$
- Calibration: $\theta_F = 0.0505$
- Data: Option-Based
- Data: Livingston Survey

Supplemental Figures

Figure A7 Term Structure of Return Expectations

See Section 2 for more details.



Figure A8 Measuring the LVIX

This figure visualizes the LVIX integral (7) in the Black-Scholes model:

$$\mathscr{L} - r = \frac{1}{P} \int_0^\infty \omega(K) f(K) dK = \frac{1}{P} \left[\int_0^F \frac{\operatorname{put}(K)}{K} dK + \int_F^\infty \frac{\operatorname{call}(K)}{K} dK \right].$$

Time and time-to-maturity dependence are omitted for simplicity. The x-axis units are volatility standard deviations from the forward price: $\frac{K/F-1}{\sigma\sqrt{\tau}}$. Parameters: P = 100, $\Delta K = 0.01$, r = 0.05, $\tau = 1$, IV = 0.20, q = 0.00. See Appendix C.1 for more details.



Figure A9 Filtered Option Prices

Panel A plots the number of observations after filters. Panel B plots the share of filtered options with positive open interest. Each bar is the annual median from daily data. This sample is the longest available for each exchange from 1990 to 2020. The black line is the unconditional median from daily data. This sample is from May 14, 2007 to April 28, 2021. Options are out-of-the-money with maturity $7 \le \tau \le 365$ days. Belgium, Finland, and the Stoxx Europe 50/600 do not have open interest data. See Appendix B.1 for more details.





Figure A10 Filtered Option Prices by Maturity

This figure plots the share of filtered options by maturity bin. Each bar is the annual share from daily data. Options are out-of-the-money with maturity $7 \le \tau \le 365$ days. The sample is the longest available for each exchange from 1990 to 2020. See Appendix B.1 for more details.



Figure A11 Minimum/Maximum Strike Price: Full Sample

This figure plots the minimum/maximum strike price by maturity bin. The left panel is the annual median from daily data. This sample is the longest available for each exchange from 1990 to 2020. The right panel is the unconditional median from daily data. This sample is from May 14, 2007 to April 28, 2021. The minimum is the 1st percentile from out-of-the-money put options. The maximum is the 99th percentile from out-of-the-money call options. The units are risk-neutral standard deviations from the forward price: $\frac{K/F-1}{\sigma\sqrt{\tau}}$. See Appendix B.1 for more details.







(Continued on the next page)

Figure A11 Minimum/Maximum Strike Price: Full Sample (Continued)



Panel C. Minimum Strike Price: $11 \le \tau \le 13$ Months-to-Maturity







Figure A12 Surface Triangulation

This figure plots the surface triangulation of option prices on four dates in the U.S. sample. The dots are observed option prices with moneyness $0.80 \le K/P \le 1.20$ and maturity $30 \le \tau \le 365$ days. See Appendix B.2 for more details.



Figure A13 Option-Based Spot and Forward Rates: Full Sample

Panel A plots contemporaneous 6-month spot rates $\tilde{\mu}_t^{(6)}$ (light blue) and 6 × 6-month forward rates $\tilde{f}_t^{(6,6)}$ (dark blue). Panel B plots instrumented 6 × 6-month forward rates $\tilde{f}_t^{(6,6)}$ (dark blue) and the corresponding realized 6-month spot rates $\tilde{\mu}_{t+6}^{(6)}$ (red). The instrument is the 2 × 1-month forward rate: see Table 3 for more details. Spot and forward rates are for option-based risk premia. Gray bands are NBER recessions. The sample is the longest available for each exchange. See Appendix C.3 for more details.





40

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20 10 2020

2020

Figure A14 Power Utility: Regression Slopes and Average Forecast Errors

This figure reports estimates from the standpoint of an unconstrained power utility investor fully invested in the market for option-based risk premia. The sample is the longest available for each exchange in the main sample. See Appendix D.1 for more details.



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