Forward Return Expectations

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Background

Well-studied set of questions:

- What is the expected (excess) return on the market?
- How does it evolve over time?
- Are there systematic errors in return predictions?

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$$p_t - d_t = \kappa - \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1}}_{\mathbb{E}_t r_{t+1} + \sum_{j=1}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1}} + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}$$

$\sum_{j=1}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1}$: *much more important for pricing!*

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$$p_t - d_t = \kappa - \underbrace{\sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1}}_{\mathbb{E}_t r_{t+1} + \sum_{j=1}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1}} + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}$$

Our focus:

- What is the **expected future equity premium**?
- How does it compare to the *actual* future equity premium $\mathbb{E}_{t+j} r_{t+j+1}$?
- Are there systematic errors in *expected* return predictions?

What We Do

1. Using options & surveys, measure return expectations at multiple horizons n (short & long):

$$\textbf{Spot rate (current):} \quad \mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n}]$$

2. From these, back out expected future return expectation (*forward return expectation*):

$$\textbf{Forward rate (future):} \quad f_t^{(n)} = \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(1)}]$$

3. Compare forward rate to realized future spot rate:

$$\textbf{Forecast error:} \quad \varepsilon_{t+n} = \mu_{t+n}^{(1)} - f_t^{(n)}$$

- Usually will subtract risk-free rate & consider spot/forward risk premium ($\tilde{\mu}_t^{(n)} = \mathbb{E}_t[r_{t,t+n} - r_{t,t+n}^f]$)
  (paper also has results for expected returns)
- Options: New theoretical results to measure forecast errors in forward rates
- Surveys: Term structure of expected returns in Livingston, Duke CFO, Vanguard

What We Do

$$
\begin{aligned}
\textbf{Spot rate:} \quad & \mu_t^{(n)} &= \mathbb{E}_t[r_{t,t+n}] \\
\textbf{Forward rate:} \quad & f_t^{(n)} &= \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(1)}] \\
\textbf{Forecast error:} \quad & \varepsilon_{t+n} &= \mu_{t+n}^{(1)} - f_t^{(n)}
\end{aligned}
$$

Measurement:

1. Option prices
   - Theory-based tools to measure forward rates & forecast errors
   - Test whether expectations are intertemporally consistent, without needing to take a stand on whether spot expected returns are themselves rational
   - Rich data...but ultimately model-based
2. Survey expectations
   - Term structure of expected returns in Livingston, Duke CFO, and Vanguard surveys
   - Simple, model-free estimates (will start with these). . .but not as rich data

What We Find

$$
\begin{aligned}
\textbf{Spot rate:} \quad & \mu_t^{(n)} &&= \mathbb{E}_t[r_{t,t+n}] \\
\textbf{Forward rate:} \quad & f_t^{(n)} &&= \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(1)}] \\
\textbf{Forecast error:} \quad & \varepsilon_{t+n} &&= \mu_{t+n}^{(1)} - f_t^{(n)}
\end{aligned}
$$

Excess countercyclicality in forward return expectations:

1. In options & all surveys, forward rates are consistently countercyclical. . .
   - When the market $\searrow \implies$ expectations of future equity premia $\nearrow$
   - Contrasts with short-horizon return extrapolation in some surveys
2. . . .and in fact **too countercyclical**
   - In bad times, investors believe expected returns will stay elevated for longer and by more than their own subsequent beliefs justify (vice versa in good times)
   - Thus excessively cyclical (and excessively volatile) forward return expectations
   - As if investors **overreact to news about risk**

A Unified View of Subjective Return Expectations

	Horizon of expectations				
[**+** : *pos. comovement with objective meas.*]	Short run	+	Forward *(this paper)*	=	Long run
Sophisticated investors					
"Mr. Market" (options) [Martin]	**+**		**+** (excess)		**+**
Asset managers [Dahlquist-Ibert]					**+**
Other professionals					
CFOs [Greenwood-Shleifer, De La O-Myers]	**–**		**+** (excess)		**+**
Prof. forecasters [Nagel-Xu]	**+** (insufficient)		**+** (excess)		**+**
Retail investors					
Vanguard investors [Giglio et al.]	**–**		**+** (excess)		**+**
Gallup survey [Greenwood-Shleifer]	**–**				

Implications

Excess cyclicality in forward return expectations helps us understand:

1. Debate on cyclicality of subjective risk premia
   - Short-run expectations differ across investor groups (pro-, a-, or countercyclical)
   - Forward expectations are *consistently* countercyclical — and excessively so
2. Excess volatility in stock prices
   - When prices are depressed, partly reflects investors expecting persistently high risk premia
   - Lower-bound estimates: 20–25% of price declines during 2008 and Covid crises; largest estimates explain the majority of the drops
3. Facts about equity term structure & inelastic demand for equities *(in paper)*
4. Models of belief formation & forecasting in different environments

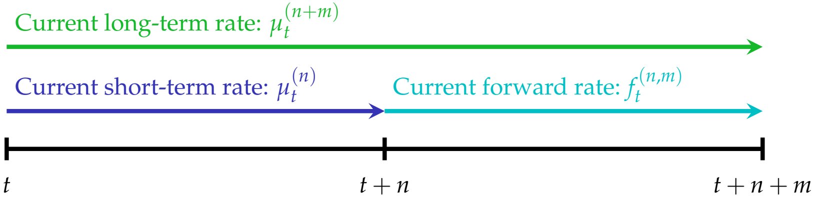
Explanations: Simple, externally calibrated model of overreaction to risk at long horizons works well

. . .and also explore how it may coexist alongside short-run return extrapolation

Roadmap

1. Introduction
2. Measuring Forward Expectations
   - Identification Challenges
   - Survey-Based Measurement
   - Price-Based Measurement
3. Empirical Tests
4. Forecast Errors: Implications and Explanations
5. Conclusions

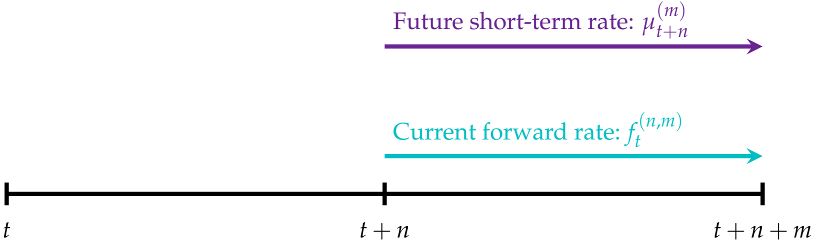
Identification Challenges



Issues:

1. How to measure spot & forward rates for risky assets
   - For risk-free Treasury term structure, rates are observable at multiple horizons

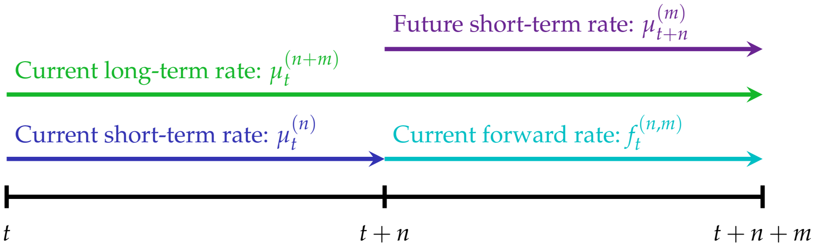
Identification Challenges



Issues:

1. How to measure spot & forward rates for risky assets
   - For risk-free Treasury term structure, rates are observable at multiple horizons
2. For rates measured from market prices, how to deal with risk premium
   - May have $f_t^{(n,m)} \neq \mathbb{E}_t[\mu_{t+n}^{(m)}]$, since price $\neq$ subjective expectation
   - Same issue arises for risk-free term structure (failure of expectations hypothesis)

Identification Challenges



Issues:

1. How to measure spot & forward rates for risky assets
2. How to deal with risk premium

Solutions:

(i) Find surveys that elicit return expectations at multiple horizons

(ii) Use a theoretically motivated set of derivatives prices

Survey-Based Measurement

Three surveys that elicit return expectations at multiple horizons:

1. Livingston survey of professional economists & forecasters, via Philly Fed (1992–2021, $2\times$/yr)
   - Median S&P 500 price expectations at 6m & 12m horizon, from which:
     - $\mu_t^{(12\text{ months})}, \mu_t^{(6\text{ months})}$
     - $f_t^{(6\text{ months})} = \mu_t^{(12\text{ months})} - \mu_t^{(6\text{ months})}$
     - $\varepsilon_{t+6\text{ months}}^{(6\text{ months})} = \mu_{t+6\text{ months}}^{(6\text{ months})} - f_t^{(6\text{ months})}$
   - When considering risk premia, use T-bill expectations at 6m & 12m horizon as well

Survey-Based Measurement

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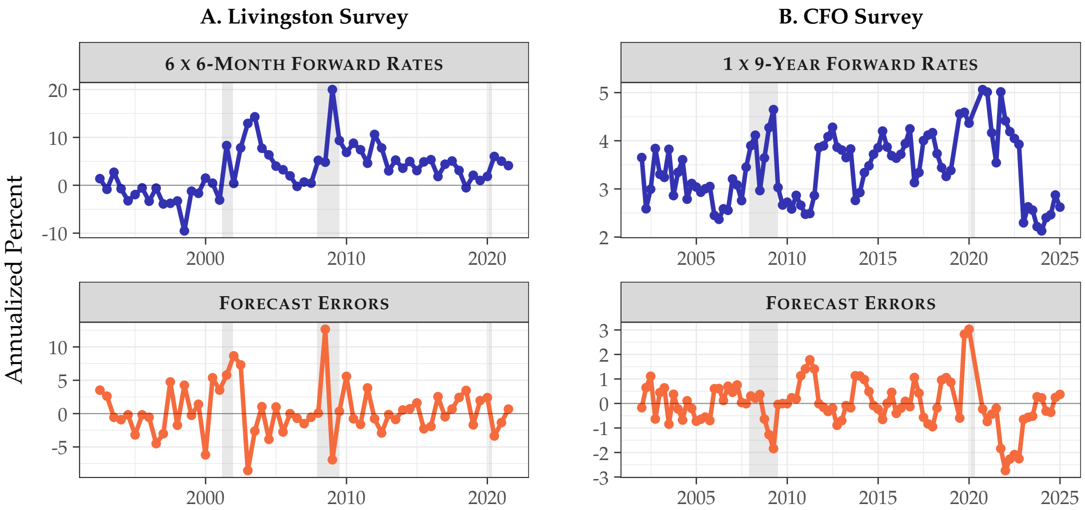
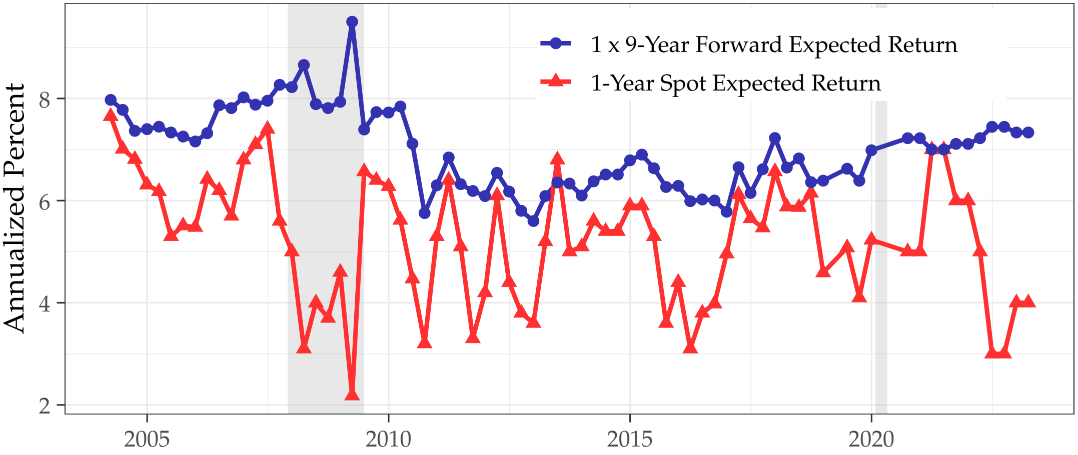
     - $\mu_t^{(12\text{ months})}, \mu_t^{(6\text{ months})}, f_t^{(6\text{m})} = \mu_t^{(12\text{m})} - \mu_t^{(6\text{m})}, \varepsilon_{t+6\text{m}}^{(6\text{m})} = \mu_{t+6\text{m}}^{(6\text{m})} - f_t^{(6\text{m})}$
2. Duke–Fed CFO survey (2001–2025, quarterly; structural survey break in 2020) Break plot
   - Mean 1y & 10y S&P 500 return expectations, from which:
     - $\mu_t^{(10\text{ years})}, \mu_t^{(1\text{ year})}$
     - $f_t^{(1\text{ year, }9\text{ years})} = \mu_t^{(10\text{ years})} - \mu_t^{(1\text{ year})}$
     - $\varepsilon_{t+1\text{ year}}^{(9\text{ years})} \approx \mu_{t+1\text{ year}}^{(10\text{ years})} \times 9/10 - f_t^{(1\text{ year, }9\text{ years})}$
   - For risk premia, subtract observed 10y & 1y yield on Treasuries
3. Vanguard survey of retail investors (2017–2024, bimonthly, $\sim$2,000 respondents)
   - Mean 1y & 10y expected returns, same construction as CFO survey

Excessively Countercyclical Forward Risk Premia



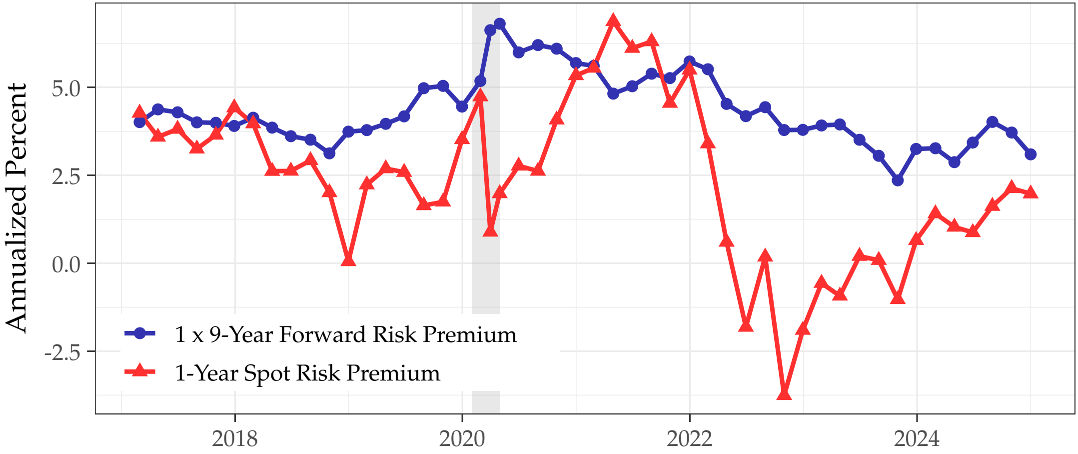
CFO Spot and Forward Rates: Importance of Long Horizon



- CFOs extrapolate past returns at short horizons, but long-horizon forecasts are (too) **countercyclical**
- Countercyclical forwards are stronger for risk premia (previous slide)

Vanguard Retail Survey: Spot and Forward Rates

Same pattern in Vanguard survey of ∼2,000 retail investors (2017–2024):



- Forecast error predictability also holds statistically, as we'll see

Price-Based Measurement for Marginal Investor Expectations

Intuition: w/ option prices, measure $\mathbb{E}$[future eq. premium] + risk prem., and risk prem. is small

- **Building block**: **LVIX** $\mathcal{L}_t^{(n)}$ (Gao & Martin 2021): Given SDF (marginal utility) $M_{t,t+n}$,

$$\underbrace{\mathbb{E}_t[r_{t,t+n}]}_{\substack{\mu_t^{(n)} \\ \textit{[what we want]}}} = \underbrace{\mathbb{E}_t[M_{t,t+n}R_{t,t+n}r_{t,t+n}]}_{\substack{\mathcal{L}_t^{(n)} \\ \textit{[observable RN expectation]}}} - \underbrace{\mathrm{Cov}_t(M_{t,t+n}R_{t,t+n}, r_{t,t+n})}_{\substack{\mathcal{C}_t^{(n)} \\ \textit{[unobservable]}}}$$

- Steps: $\mathbb{E}_t[M_{t,t+n}R_{t,t+n}] = 1 \Rightarrow \mu_t = \mathbb{E}_t[r_{t,t+n}] = \mathbb{E}_t[r_{t,t+n}]\mathbb{E}_t[M_{t,t+n}R_{t,t+n}]$
- $\mathcal{C}_t^{(n)} = 0$ under log utility ($MR = 1$)...otherwise introduces unobservable contamination
- Gao & Martin: $\mathcal{C}_t^{(n)} \leqslant 0$...but what about for fwd rate $f_t^{(n,m)} = \mathcal{L}_t^{(n+m)} - \mathcal{L}_t^{(n)} + \mathcal{C}_t^{(n)} - \mathcal{C}_t^{(n+m)}$?
- **Key insight:** Covariance terms largely cancel when measuring **forecast errors** $\varepsilon_{t+n}^{(m)} = \mu_{t+n}^{(m)} - f_t^{(n,m)}$
- Option-based expected returns may not be good predictors of realized returns...

  ...but they should be **consistent over time**

The Log-Normal Case: Result

Observable forecast-error proxy:

$$\widehat{\varepsilon}_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} - \mathcal{L}_t^{(n+m)} + \mathcal{L}_t^{(n)}$$

Result 1 (*Log-Normal Identification*)

For a general SDF $M_{t,t+n}$, assuming $M_{t,t+n}$, $R_{t,t+n}$ are jointly log-normal:

$$\mathbb{E}_t\left[\widehat{\varepsilon}_{t+n}^{(m)}\right] = \mathbb{E}_t\left[\varepsilon_{t+n}^{(m)}\right] - \mathrm{Cov}_t(M_{t,t+n}R_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}])$$

- Covariance term now relates to pricing of *discount-rate risk*, rather than *realized-return* risk
- Likely much smaller than previous term: expected returns are much less volatile than realized returns
- Can be disciplined empirically or theoretically
- As long as covariance is small, can use LVIX to measure fwd-rate forecast errors & test $\mathbb{E}_t[\varepsilon_{t+n}^{(m)}] = 0$
- Basic idea of proof: $MR_{t,t+n}$ is orthogonal to unexpected component of $r_{t+n,t+n+m}$

The General Case: Result

Define forecast-error proxy and expected-return proxy:

$$\widehat{\varepsilon}_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} - \mathcal{L}_{t}^{(n+m)} + \mathcal{L}_{t}^{(n)}$$

$$\widehat{\mu}_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} + r_{t+n,t+n+m}^{f}$$

Result 2 (*Generalized Identification*)

For any SDF $M_{t,t+n}$,

$$\mathbb{E}_t\left[\widehat{\varepsilon}_{t+n}^{(m)}\right] = \mathbb{E}_t\left[\varepsilon_{t+n}^{(m)}\right] - \mathrm{Cov}_t\left(M_{t,t+n}R_{t,t+n}, \widehat{\mu}_{t+n}^{(m)}\right)$$

- Intuition from log-normal case carries over, with $\mathbb{E}_{t+n}[r_{t+n,t+n+m}]$ replaced by $\widehat{\mu}_{t+n}^{(m)}$
- LVIX-based $\widehat{\mu}_{t+n}^{(m)}$ is closely related to $\mathbb{E}_{t+n}[r_{t+n,t+n+m}]$ …but $\widehat{\mu}_{t+n}^{(m)}$ is directly observable
- Main specification: $\widehat{\mu}_{t+n}^{(m)}$ is $\frac{1}{10}$ as volatile as realized return $r_{t+n,t+n+m}$

  $\Longrightarrow$ unobserved covariance likely smaller for forecast errors than for spot rates

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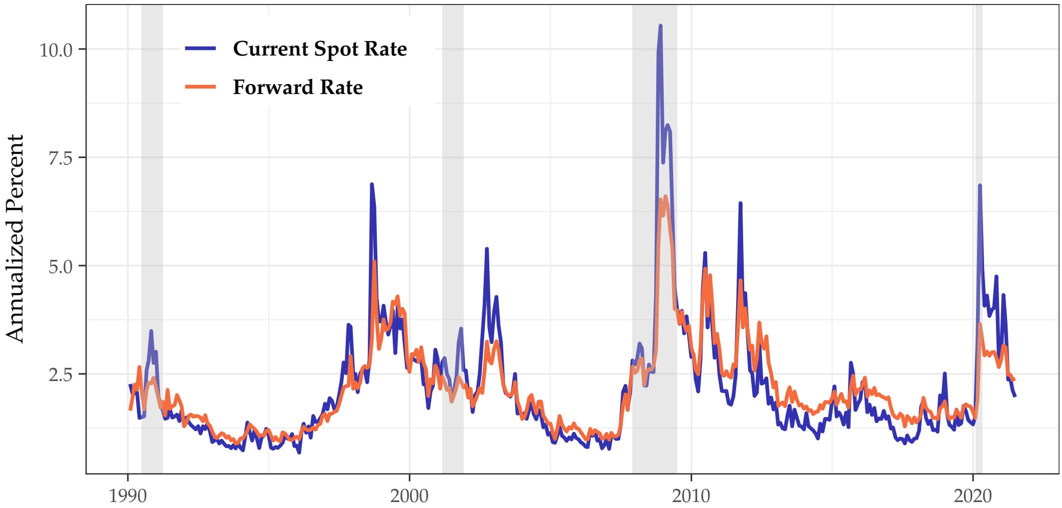
Data and Measurement: Options

Data:

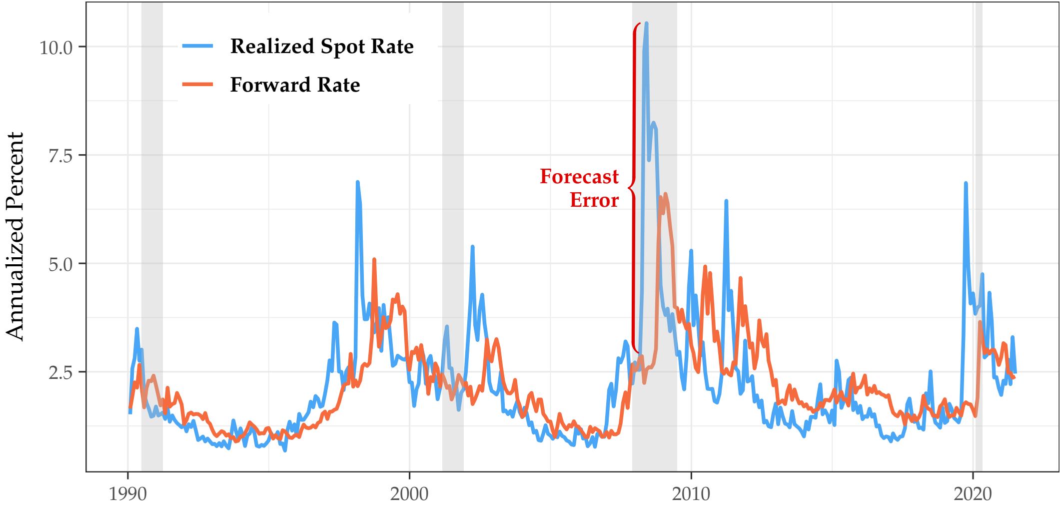
- Main data: Global panel of index options from OptionMetrics (monthly data, standard filters)
  - For U.S. sample: 1990–2021
  - For international sample: Consider 10 major indices, with data since at least 2006
- Baseline: 6-month horizon, 6 months forward ($n = m = 6$)
- Measuring LVIX: Integrate price$(K)/K$ following Carr & Madan (2001)
  [extrapolate past observed strikes using a bunch of different methods]

- Calculate integral a bunch of different ways
- First: Simplify by working under log assumption, so LVIX $\Longrightarrow$ spot & forward rates

Estimates: Contemporaneous U.S. Spot and Forward Rates



Estimates: Realized U.S. Spot and Forward Rates



Do Forward Rates Predict Future Spot Rates?

Mincer–Zarnowitz Regressions for Future Spot Rates

$$\tilde{\mu}^{(6)}_{t+6} = \beta_0 + \beta_1 \tilde{f}^{(6,6)}_t + \epsilon_{t+6}$$

	(1) U.S.	(2) Ex-U.S.	(3) All
$f^{(6,6)}_t$	0.67***	0.55***	0.56***
	(0.096)	(0.056)	(0.055)
Intercept	0.74***		
	(0.28)		
Country FEs	✗	✓	✓
p-value: $\beta_1 = 1$	0.003	0.000	0.000
Obs.	378	1,849	2,227
R^2	0.22	0.21	0.22
Within R^2	—	0.14	0.15

SEs in (1) are heteroskedasticity and autocorrelation-robust [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Substantial predictive power…
  …but $\beta_1 \neq 1$, so forward rates overshoot future spot rates
- The market **qualitatively** understands variation in the equity premium, but **quantitatively** significant excess forward volatility
- What if β_1 estimate is downwardly biased due to measurement error?
- Consider IV using shorter-term forward rate $f^{(2,1)}_t$ (shorter horizons likely better measured)

Do Forward Rates Predict Future Spot Rates?

Instrumented Mincer–Zarnowitz Regressions for Spot Rates

$$\tilde{\mu}^{(6)}_{t+6} = \beta_0 + \beta_1 \tilde{f}^{(6,6)}_t + \epsilon_{t+6}, \quad \tilde{f}^{(6,6)}_t = \pi_0 + \pi_1 \tilde{f}^{(2,1)}_t + \eta_t$$

	(1) U.S.	**(2)** Ex-U.S.	**(3)** All
$f^{(6,6)}_t$	0.73*** (0.062)	0.69*** (0.078)	0.70*** (0.074)
Intercept	0.59*** (0.13)		
Country FEs	✗	✓	✓
p-value: $\beta_1 = 1$	0.018	0.004	0.003
Obs.	378	1,849	2,227
R^2	0.22	0.20	0.22
Within R^2	—	0.13	0.14

SEs in (1) are heteroskedasticity and autocorrelation-robust [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Forward rate ↗ by **1%** $\Longrightarrow$ future spot rate ↗ by ∼**0.7%**
- Forward rates explain ∼**20%** of the variation in future spot rates
- Again: The market **qualitatively** understands variation in the equity premium, but **quantitatively** significant excess forward volatility

Average Forecast Errors Are Close to Zero

Average Forecast Errors Across Countries

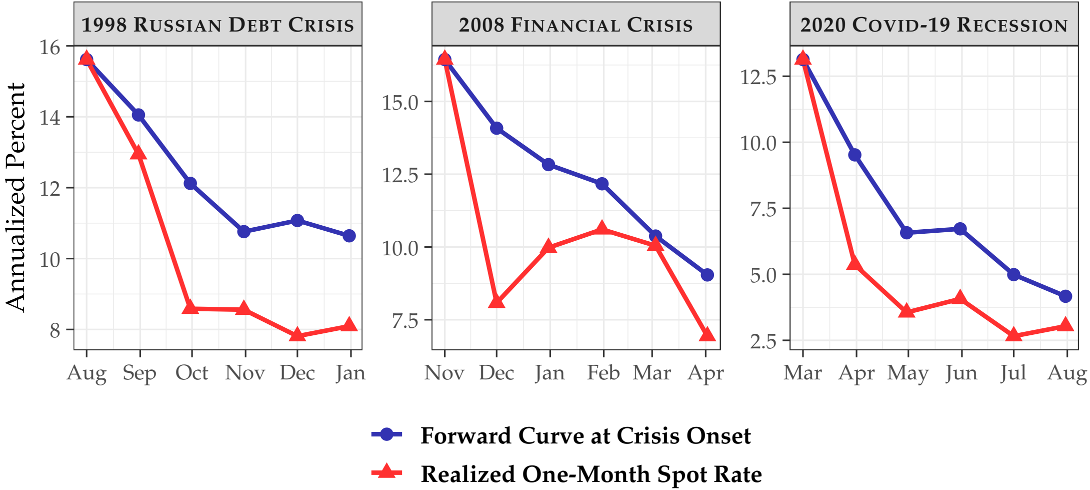
$$\tilde{\varepsilon}^{(6)}_{t+6} = \tilde{\mu}^{(6)}_{t+6} - \tilde{f}^{(6,6)}_{t}$$

	(1) U.S.	**(2)** Ex-U.S.	**(3)** All
Average	0.021	0.20	0.17
	(0.15)	(0.11)	(0.11)
Obs.	378	1,849	2,227

SEs in (1) are HAR [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Not just statistically insignificant, but effectively zero: $\bar{\varepsilon} \leqslant 20$ bps annualized
- But average of zero masks substantial predictability. . .

Option-Based Forward and Realized Spot Rates in Crises



. . . and pattern holds systematically in both options & surveys.

Forward Rates as Predictors of Forecast Errors

Predictable Forecast Errors

Regressions of Forecast Errors on 2×1 Forward Rate

$$\tilde{\varepsilon}^{(6)}_{t+6} = \beta_0 + \beta_1 \tilde{f}^{(2,1)}_t + e_{t+6}$$

	(1) U.S.	**(2)** Ex-U.S.	**(3)** All
$f_t^{(2,1)}$	-0.17** (0.066)	-0.16** (0.049)	-0.16*** (0.047)
Intercept	0.39* (0.23)		
Country FEs	✗	✓	✓
Obs.	378	1,849	2,227
R^2	0.04	0.04	0.04
Within R^2	—	0.03	0.03

SEs in (1) are HAR [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

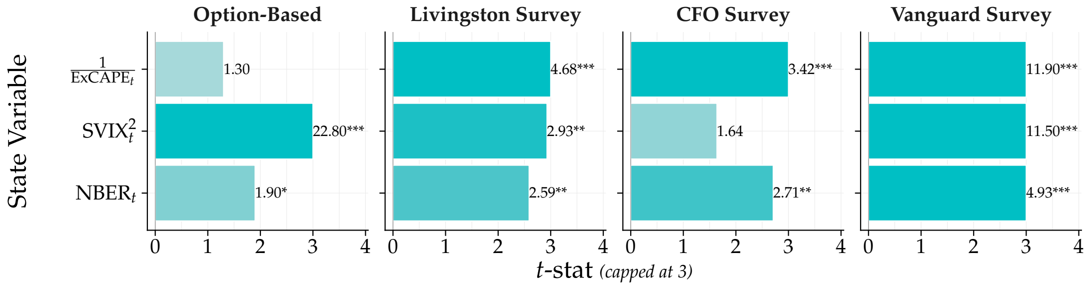
- Forward rates again overshoot future spot rates
- Errors are also predictable in Coibion–Gorodnichenko regressions using forward-rate *revisions*
- Is this consistent with overreaction?
  It depends: Overreaction to what?
  - News about risk/risk premia: Yes
  - Past returns: Wrong direction
  - Consistent excess cyclicality

Consistent Evidence From Options and Surveys

	Expectations Measured by:			
	Options	Livingston Survey	CFO Survey	Vanguard Survey
Panel A: Predictability of Spot Rates $\left(\tilde{\mu}_{t+1} = \beta_0 + \beta_1 \tilde{f}_t + e_{t+1}\right)$				
β_1	0.67***	0.81*	0.35***	0.43**
R^2	0.22	0.56	0.59	0.13
Panel B: Predictability of Forecast Errors $\left(\tilde{\varepsilon}_{t+1} = \beta_0 + \beta_1 \tilde{f}_t + e_{t+1}\right)$				
β_1	-0.17**	-0.19*	-0.65***	-0.57**
R^2	0.04	0.05	0.48	0.21

U.S. data. Options/Livingston: 6×6m. CFO/Vanguard: $1y \times 9y$. SEs are HAR [Lazarus et al. (2018)], and significance stars in Panel A are for test of $\beta_1 = 1$.

Robustly Countercyclical Forward Risk Premia



- All forward risk premia comove with standard countercyclical indicators: inverse valuation ratio (1/ExCAPE), option-implied volatility (SVIX), and NBER recession indicator
- So for all investor groups, expected future equity premia go up concurrently in bad times, and by too much relative to future views on actual equity premium

Roadmap

1. Introduction
2. Measuring Forward Expectations
3. Empirical Tests
4. Forecast Errors: Implications and Explanations
   - Implications & Excess Volatility in Crises
   - Explanations & A Simple Model
5. Conclusions

Implications

Excess cyclicality in forward return expectations helps us understand:

1. Debate on cyclicality of subjective risk premia
   - Short-term return expectations sometimes appear procyclical [Greenwood & Shleifer 2014], acyclical [Nagel & Xu 2022], or countercyclical [Dahlquist & Ibert 2024]
   - But long-horizon forward expectations are countercyclical across all data sources (excessively so)
   - Disagreement stems in part from differences in horizon
   - Our focus on forward rates eliminates disagreement across data sources

Implications

Excess cyclicality in forward return expectations helps us understand:

2. Excess volatility in stock prices
   - When prices are depressed, this partly reflects investors expecting persistently high risk premia
   - If investors didn't overestimate persistence, would see more modest fluctuations in prices
   - Next slides: Quantification

How Significant Are Forecast Errors for Price Variation?

$$p_t - d_t = \kappa - \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t r_{t+j+1} + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}$$

How Significant Are Forecast Errors for Price Variation?

$$p_t - d_t = \kappa - \mathbb{E}_t r_{t+1} - \sum_{j=1}^{\infty} \rho^j f_t^{(j,1)} - \underbrace{RF_t}_{\text{discounted risk-free rates}} + \sum_{j=0}^{\infty} \rho^j \, \mathbb{E}_t \Delta d_{t+j+1}$$

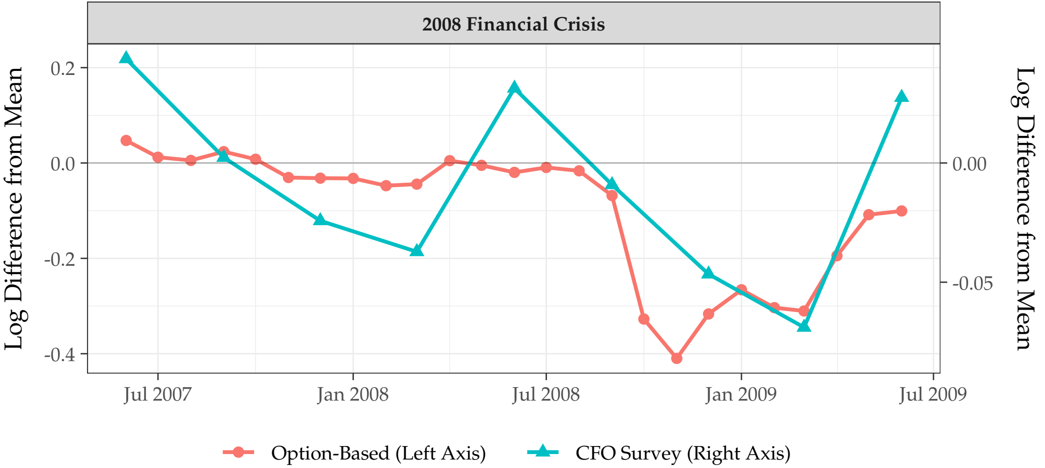
Break $f_t^{(j,1)}$ into:

$$f_t^{(j,1)} = \underbrace{\mathbb{E}_t[\mu_{t+j}^{(1)}]}_{\text{expected spot rates}} - \underbrace{\mathbb{E}_t[\varepsilon_{t+j}^{(1)}]}_{\text{predictable forecast errors}}$$

Call $\mathcal{E}_t \equiv \sum_{j=1}^{\infty} \rho^j \mathbb{E}_t[\tilde{\varepsilon}_{t+j}^{(1)}]$ discounted forecast errors, which have 1:1 impact on prices. Two measures:

1. **Options:** Use baseline predicted forecast errors
   - Estimate decay parameter in $\mathbb{E}_t[\varepsilon_{t+j+1}^{(1)}] = \phi^j \mathbb{E}_t[\varepsilon_{t+j}^{(1)}] \Longrightarrow \widehat{\phi} \approx 1$ *(using longer-dated SX5E data)*
   - Using this, $\widehat{\mathcal{E}}_t^{\text{option}} = \frac{\rho}{1-\rho} \mathbb{E}_t[\varepsilon_{t+1}^{(1)}]$
2. **Surveys:** CFO (2008), Vanguard (2020)
   - No scaling required: estimate first 9 years of forecast errors from predictable component of $\varepsilon_{t+1}^{(9)}$
   - $\widehat{\mathcal{E}}_t^{\text{survey}} = \sum_{j=1}^{9} \rho^j \mathbb{E}_t[\varepsilon_{t+j}^{(1)}] \leq \mathcal{E}_t$, with $\mathbb{E}_t[\varepsilon_{t+j}^{(1)}] = \mathbb{E}_t[\varepsilon_{t+1}^{(9)}]$ annualized

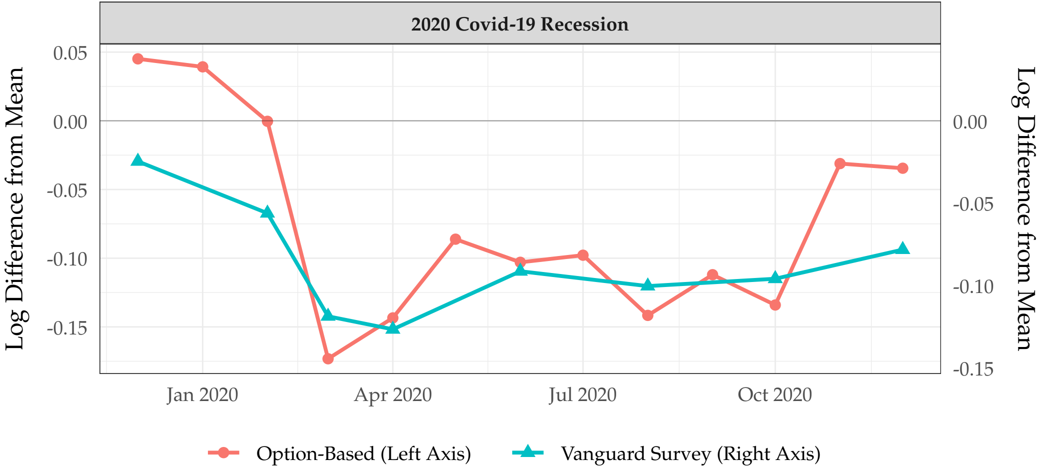
Price Impact of Forecast Errors in Crises



Overestimated future risk premia account for 11% (survey) to 35% (options) of 43% ’08 price drop

Less strong outside crises, but big impact in bad times ⇒ contributor to excess vol.

Price Impact of Forecast Errors in Crises



Overestimated future risk premia account for 9% (survey) to 22% (options) of 21% ’20 price drop

Less strong outside crises, but big impact in bad times ⇒ contributor to excess vol.

Further Implications

Excess cyclicality in forward return expectations helps us understand:

3. Facts about equity term structure from dividend claims
   - Risk premia higher than expected $\Longrightarrow$ neg. realized returns, especially for long-duration assets
   - Can generate downward sloping equity term structure on average [Binsbergen, Brandt, Koijen 2012] & upward sloping term structure during bad times (countercyclical variation) [Gormsen 2021]
   - Illustrative quantification Plot
4. More speculatively: Inelastic demand for equities [Gabaix & Koijen 2022]
   - Two possibilities given price drop: Large change in return expectations at short horizons, or moderate change in return expectations at all horizons (our finding)

   $\Longrightarrow$ shouldn't see big increase in current portfolio weight, partially resolving puzzle

Explaining Forecast Errors

Paper considers a few alternative explanations for forecast errors:

1. RE + risk premium Details
   - Price of discount-rate risk must be highly volatile and countercyclical for this to work
   - E.g., if $\text{Corr}_t(r_{t+1}, \mathbb{E}_{t+1} r_{t+2}) = -1$ and negative correlation condition (Gao & Martin 2021) $\Longrightarrow$ **relevant SDF-related covariance can't change sign**
   - Doesn't work for survey evidence
2. Aggressive policy responses that were **rationally unexpected ex ante** Details
   - Ex post monetary policy path surprises are significantly related to ex post forecast errors. . .
   - . . .but explain $\leqslant 10\%$ of their variation
3. **Expectation errors**
   - Simple model: Extrapolation of current level of risk for long-term mean $\Rightarrow$ fwd rates overreact
   - Short-rate bias: Incorrect map from risk to current short rate (allowing short-hor. ret. extrap.)
   - Matches both excess cyclicality in fwd rates & diff. spot-rate cyclicality across investors

A Simple Model of Expectation Errors

- Builds on Hirshleifer, Lim, Teoh (2015) and similar to diagnostic model (BGLS 2019) for fwd rates
- Countercyclical state variable x_t — think of as market variance — follows AR(3)
- Short rate: $\mathbb{E}_t[\mu_t^{(3)}] = \alpha_0 + \alpha_1 x_t$ with $\alpha_1 > 0$ (countercyclical)
- **Bias 1 — Forward rate bias θ_F:** Investors overreact to news about long-run mean $\bar{x}$:

$$\mathbb{E}_t^{\theta}[\bar{x}] = \mathbb{E}_{t-1}^{\theta}[\bar{x}] + \theta_F \left(x_t - \mathbb{E}_{t-1}^{\theta}[\bar{x}] \right)$$

- **Bias 2 — Short rate bias θ_S:** Investors misperceive current short rate:

$$\mathbb{E}_t^{\theta}\left[\mu_t^{(3)}\right] = \mathbb{E}_t\left[\mu_t^{(3)}\right] + \alpha_1 \theta_S \, e_t$$

- θ_F governs excess cyclicality in forwards (**same for all investors**)
- θ_S governs cyclicality of spot rates (**differs across investor groups**)

Model vs. Data and Broader Lessons

- Calibrate x_t to option-implied variance in U.S. sample, and calibrate mapping to spot rates
- $\theta_F = 0.0505$ matches both MZ and error-predictability slopes for options *(similar to past estimates)*
  Plots
- Model also explains heterogeneous spot-rate cyclicality:
  - $\theta_S < 0$: procyclical spot rates (e.g., CFOs, retail — extrapolation)
  - $\theta_S \approx 0$: acyclical spot rates (e.g., professional forecasters)
  - $\theta_S > 0$: countercyclical spot rates (e.g., "Mr. Market")
- **Connection to broader literature on belief formation:**
  - Our results: Overreaction to news about risk premia (low persistence, little long-run news)
  - Risk-free rates have high persistence & **underreaction** there [Farmer, Nakamura, Steinsson 2024]
  - One unification: Overinference from weak signals, underinference from strong [Augenblick, Lazarus, Thaler 2025]

Implied Persistence in Equity-Premium and T-Bill Forecasts

Regressions of Forecasted and Realized Values on Current Values

$$\mathbb{E}_t[\mu_{t+n}] = \alpha_{\text{subj}} + \hat{\rho}\,\mu_t + e_{t,\text{subj}}, \quad \mu_{t+n} = \alpha_{\text{obj}} + \rho\,\mu_t + e_{t,\text{obj}}$$

	Eq. Prem. Forecasts		T-Bill Forecasts
	Options	Livingston Survey	SPF Survey
$\hat{\rho}$	0.65	0.68	0.84
	[0.02]	[0.08]	[0.04]
ρ	0.48	0.64	0.91
	[0.05]	[0.10]	[0.03]

- Overreaction for equity-premium forecasts, underreaction for yield forecasts
- Appears to be **partial** shrinkage to moderate persistence. Consistent with forecasters understanding directional impact of news, but not knowing precise strength.

Roadmap

1. Introduction
2. Measuring Forward Expectations
3. Empirical Tests
4. Forecast Errors: Implications and Explanations
5. Conclusions

Final Notes

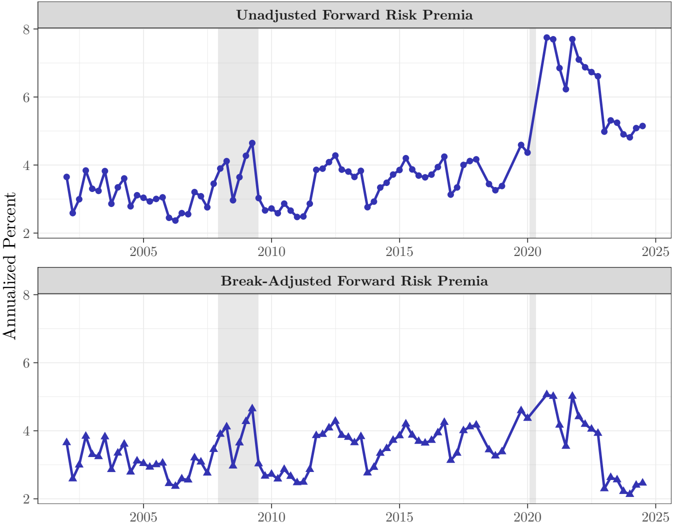
Summary:

- Introduce new methods to measure term structure of expected equity premia
- Robust evidence of excess countercyclicality in forward return expectations
- Investors consistently overestimate how long their own expected returns will stay elevated during bad times, and vice versa during good times
- Consistent across investor groups: options and three surveys (Livingston, CFO, Vanguard)

Thank you!

Appendix

CFO Forward Rates: 2020 Break from Survey Admin. Change



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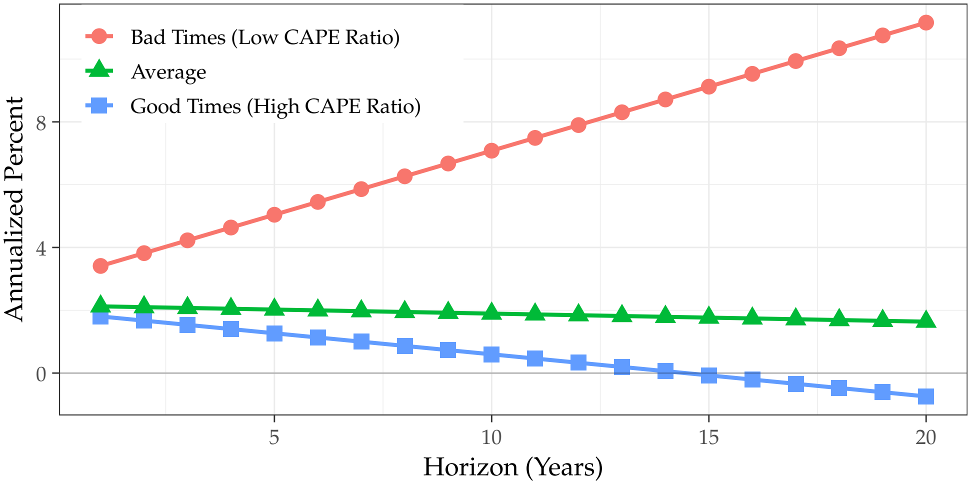
Forecast Errors: Estimated Effect on Returns for Equity Term Structure



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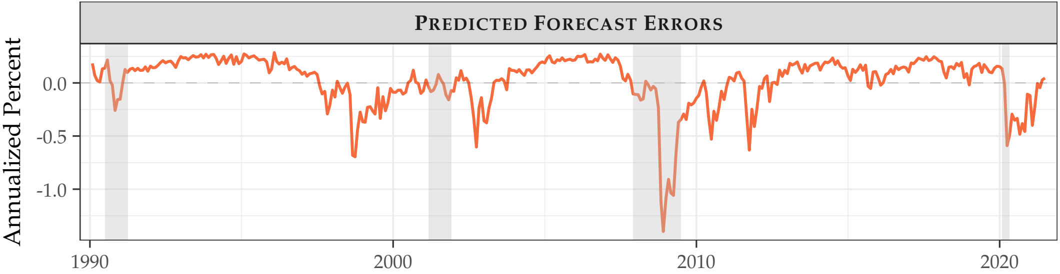
Can Forecast Errors From Price-Based Measure Be Rationalized?

$$\mathbb{E}_t[\widehat{\varepsilon}_{t+n}^{(m)}] = \mathbb{E}_t[\varepsilon_{t+n}^{(m)}] - \underbrace{\text{Cov}_t(MR_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}])}_{\varsigma_t}$$

What conditions do we need on ς_t in order for **expectation errors** $\mathbb{E}_t[\varepsilon_{t+n}^{(m)}]$ to be unpredictable?

Must have $-\varsigma_t$ take same sign as pred. forecast errors:



Main challenge: Small on average, but must flip signs dramatically (− in good times, + in bad).

Can Forecast Errors From Price-Based Measure Be Rationalized?

$$\mathbb{E}_t[\widehat{\varepsilon}^{(m)}_{t+n}] = \mathbb{E}_t[\varepsilon^{(m)}_{t+n}] - \underbrace{\text{Cov}_t(MR_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}])}_{\varsigma_t}$$

What conditions do we need on ς_t in order for **expectation errors** $\mathbb{E}_t[\varepsilon^{(m)}_{t+n}]$ to be unpredictable?

- For simplicity: Take $n = m = 1$, and assume SDF and returns are jointly log-normal
- Then $\varsigma_t > 0$ (as needed in bad times) if and only if:

  $$SR_t(-\mu_{t+1}) > -\rho_t(r,\mu)\sigma_t(r),$$

  where SR_t is Sharpe ratio on claim to next period's negative equity premium (low payoff is bad)
- Correlation $\rho_t(r,\mu)$ likely to be negative; for illustration, set it to -1
- Then SR_t must vary *more than* $\sigma_t(r)$ for ς_t to flip signs
- One calibration: Go back to log utility (likely to be conservative for time variation in σ_t), and estimate $\sigma_t(r)$ from options

Can Forecast Errors From Price-Based Measure Be Rationalized?

$$\mathbb{E}_t[\widehat{\varepsilon}^{(m)}_{t+n}] = \mathbb{E}_t[\varepsilon^{(m)}_{t+n}] - \underbrace{\text{Cov}_t(MR_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}])}_{\varsigma_t}$$

What conditions do we need on ς_t in order for **expectation errors** $\mathbb{E}_t[\varepsilon^{(m)}_{t+n}]$ to be unpredictable?

- SR_t must vary *more than* $\sigma_t(r)$ for ς_t to flip signs
- One calibration: Go back to conservative log utility case, and estimate $\sigma_t(r)$ from options. **Results for conditional volatility of 6-month market return:**

Can Forecast Errors From Price-Based Measure Be Rationalized?

$$\mathbb{E}_t[\widehat{\varepsilon}_{t+n}^{(m)}] = \mathbb{E}_t[\varepsilon_{t+n}^{(m)}] - \underbrace{\text{Cov}_t(MR_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}])}_{\varsigma_t}$$

What conditions do we need on ς_t in order for **expectation errors** $\mathbb{E}_t[\varepsilon_{t+n}^{(m)}]$ to be unpredictable?

- SR_t must vary *more than* $\sigma_t(r)$ for ς_t to flip signs
- Further, given $\rho_t(r, \mu) = -1$, ς_t **cannot** flip signs if mNCC [Gao & Martin (2021), Assumption 2] holds
  - $\rho_t(r, \mu) = -1 \implies \varsigma_t$ is scaled version of their covariance term $\mathcal{C}_t^{(n)}$
  - If $\mathcal{C}_t^{(n)} \leqslant 0$ (mNCC), then $\varsigma_t \leqslant 0$
- More generally, difficult to get both average errors (small) *and* time variation (large) right
- Paper has one illustration varying risk aversion γ

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Unexpected Policy Shocks: Significant, But Limited

Regressions of Forecast Errors on Ex Post Policy Shocks

$$\varepsilon^{(6)}_{t+6} = \beta_0 + \beta_1 \left(\sum_{h=1}^{6} \text{MPS}_{t+h} \right) + e_{t+6}$$

	Explained Variation (R^2)	
	Risk-Free	Risk Premium
BERNANKE-KUTTNER (2005)		
Fed Funds Rate Change	0.40***	0.09**
Fed Funds Rate Surprise	0.01	0.07***
GURKAYNAK-SACK-SWANSON (2005)		
Target Shock	0.05***	0.03*
Path Shock	0.26	0.06**
NAKAMURA-STEINSSON (2018)		
Fed Funds Rate Shock	0.03	0.03**
Policy News Shock	0.32***	0.10**

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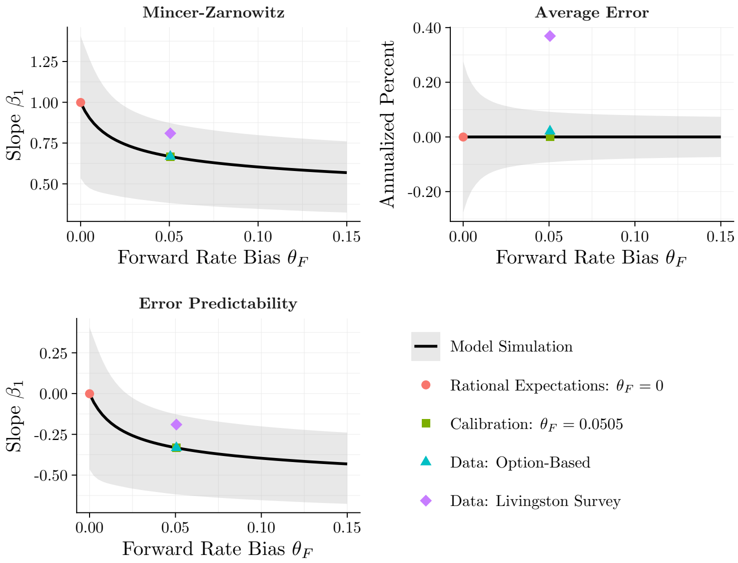
Model vs. Data: Estimates



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A Trilemma for Expectation Errors

$$p_t - d_t = \kappa - \mathbb{E}_t r_{t+1} - \underbrace{\sum_{j=1}^{\infty} \rho^j f_t^{(j,1)}}_{\mathcal{F}_t} + \underbrace{\sum_{j=0}^{\infty} \rho^j \mathbb{E}_t \Delta d_{t+j+1}}_{CF_t} - RF_t$$

- Use $\widetilde{\cdot}$ to denote **expectation error wedge** (deviation from RE economy):

$$\text{var}\left(\widetilde{p_t - d_t}\right) = \text{var}\left(\widetilde{\mathcal{F}}_t\right) + \text{var}\left(\widetilde{CF}_t\right) - 2\,\text{cov}\left(\widetilde{\mathcal{F}}_t, \widetilde{CF}_t\right)$$

- Have to choose between **two of three**:
  1. Volatile expectation errors for cash flows and/or returns
  2. Volatile price-dividend ratio relative to RE
  3. Positive comovement between fundamental and return expectation errors