Excess Persistence in Return Expectations

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Background

Well-studied set of questions:

- What is the expected excess return on the market?
- How does it evolve over time?
- Are there systematic errors in return predictions?
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\[ p_t - d_t = \kappa - \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t r_{t+j+1} + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t \Delta d_{t+j+1} \]

\[ \mathbb{E}_t r_{t+1} + \sum_{j=1}^{\infty} \rho^j \mathbb{E}_t r_{t+j+1} \]

much more important for pricing!
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\[ p_t - d_t = \kappa - \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t r_{t+j+1} + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t \Delta d_{t+j+1} \]

Our focus:

- What is the expected future equity premium?
- How does it compare to the actual future equity premium \( \mathbb{E}_{t+j} r_{t+j+1} \)?
- Are there systematic errors in expected return predictions?
What We Do

1. Measure equity premium at multiple horizons $n$ (using options or surveys):

   **Spot rate:**
   \[
   \mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n} - r_{t,t+n}^f]
   \]

2. Calculate expected future equity premium:

   **Forward rate:**
   \[
   f_t^{(n)} = \mu_{t+1}^{(n)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+1}^{(1)}]
   \]

3. Compare forward rate to realized future spot rate:

   **Forecast error:**
   \[
   \varepsilon_{t+n} = \mu_{t+n}^{(1)} - f_t^{(n)}
   \]
What We Do

Spot rate: \[ \mu_t^{(n)} = \mathbb{E}_t [r_{t,t+n} - r_{t,t+n}^f] \]

Forward rate: \[ f_t^{(n)} = \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t [\mu_{t+n}^{(1)}] \]

Forecast error: \[ \varepsilon_{t+n} = \mu_{t+n}^{(1)} - f_t^{(n)} \]

Measurement:

1. Option prices
   - Measurement of log equity premium
   - Forecast errors identified under much weaker conditions than expected returns themselves
   - We can test whether expectations are intertemporally consistent, without needing to take a stand on whether spot expected returns are themselves rational
   - Rich data...but ultimately model-based

2. Survey expectations
   - Term structure of expected returns in Livingston and Duke-CFO survey
   - Model-free tests...but not as rich data
What We Find

Spot rate: \( \mu_t^{(n)} = \mathbb{E}_t[r_{t,t+n} - r_{t,t+n}^f] \)

Forward rate: \( f_t^{(n)} = \mu_t^{(n+1)} - \mu_t^{(n)} = \mathbb{E}_t[\mu_{t+n}^{(1)}] \)

Forecast error: \( \varepsilon_{t+n} = \mu_{t+n}^{(1)} - f_t^{(n)} \)

Excess persistence in return expectations:

1. In options & surveys, forward rates are countercyclical . . .
   ▶ When the market \( \downarrow \) \( \implies \) expectations of future equity premia \( \uparrow \)
   ▶ Contrasts with short-horizon extrapolation in some surveys [Greenwood & Shleifer 2014]

2. . . .and in fact too countercyclical
   ▶ In bad times, investors believe expected returns will stay elevated for longer and by more than their own subsequent beliefs justify
   ▶ This is what we refer to as excess persistence in return expectations
Illustration: Option-Based Forward and Realized Spot Rates in Crises

**1998 Russian Debt Crisis**

- Annualized Percent:
  - Forward rate at crisis onset
  - Realized one-month spot rate

**2008 Financial Crisis**

- Annualized Percent:
  - Forward rate at crisis onset
  - Realized one-month spot rate

**2020 COVID-19 Recession**

- Annualized Percent:
  - Forward rate at crisis onset
  - Realized one-month spot rate
### Summary of Evidence

<table>
<thead>
<tr>
<th></th>
<th>Options</th>
<th>Livingston Survey</th>
<th>CFO Survey</th>
</tr>
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<tbody>
<tr>
<td><strong>Panel A. Predictability in Spot Rates</strong> ((\mu_{i,t+1} = \beta_0 + \beta_1 f_{i,t} + e_{i,t+1}))</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(\beta_1)</td>
<td>0.88</td>
<td>0.68</td>
<td>0.63</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.71</td>
<td>0.38</td>
<td>0.46</td>
</tr>
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<td><strong>Panel B. Predictability of Forecast Errors</strong> ((e_{i,t+1} = \beta_0 + \beta_1 f_{i,t} + e_{i,t+1}))</td>
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</tr>
<tr>
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<td>-0.19</td>
<td>-0.15</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.06</td>
<td>0.06</td>
<td>0.03</td>
</tr>
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<td><strong>Panel C. Cyclical Variation in Forward Rates and Forecast Errors</strong></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>(\rho (f_{i,t}, 1/CAPE_t))</td>
<td>0.04</td>
<td>0.42</td>
<td>0.21</td>
</tr>
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Implications

Excess persistence in expected returns helps us understand:

1. Excess volatility in stock prices
   - When prices are depressed, this partly reflects investors expecting persistently high risk premia
   - If investors didn’t overestimate persistence, would see more modest fluctuations in prices (about 50% less during 2008 crisis, nearly 100% less during Covid crash)

2. Inelastic demand for equities [Gabaix & Koijen 2022]
   - Puzzle: Why investors change weight in equities so modestly in response to change in stock prices
   - Partial resolution: If price drop leads to increases in expected returns mainly at long horizons, shouldn’t see big increase in portfolio weight

3. Facts about equity term structure

4. Debate on cyclicality of subjective risk premia
Roadmap

1. Introduction

2. Price-Based Measurement of Expectations: Theory

3. Evidence from Price-Based Expectations

4. Evidence from Survey-Based Expectations

5. Explaining Forecast Errors

6. Implications and Conclusions
Setting and Identification Challenge

- Representative agent (“the market”). . .or any unconstrained trader fully invested in the market

- Building block: LVIX $\mathcal{L}_t^{(n)}$ (Gao & Martin 2021):

\[
\mathbb{E}_t[r_{t,t+n} - f_{t,t+n}^f] = \mathbb{E}_t[M_{t,t+n}R_{t,t+n}r_{t,t+n} - f_{t,t+n}^f] - \text{Cov}_t(M_{t,t+n}R_{t,t+n}, r_{t,t+n})
\]

- $\mathcal{L}_t^{(n)}$: Observable from options

- $C_t^{(n)} = 0$ under log utility ($MR = 1$). . .otherwise introduces unobservable contamination

- Gao & Martin argue $C_t^{(n)} \leq 0$ . . .but what about for fwd rate $f_{t}^{(n,m)} = \mathcal{L}_t^{(n+m)} - \mathcal{L}_t^{(n)} + C_t^{(n)} - C_t^{(n+m)}$?

- Key insight: Covariance terms largely cancel when measuring forecast errors $\varepsilon_{t+n}^{(m)} = \mu_{t+n}^{(m)} - f_{t}^{(n,m)}$

- Option-based expected returns may not be good predictors of realized returns. . .

  . . . but they should predict themselves
The Log-Normal Case: Result

Observable forecast-error proxy:

$$\hat{\varepsilon}_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} - \mathcal{L}_{t}^{(n+m)} + \mathcal{L}_{t}^{(n)}$$

**Result 1 (Log-Normal Identification)**

For a general SDF $M_{t,t+n}$, assuming $M_{t,t+n}$, $R_{t,t+n}$ are jointly log-normal:

$$\mathbb{E}_t \left[ \hat{\varepsilon}_{t+n}^{(m)} \right] = \mathbb{E}_t \left[ \varepsilon_{t+n}^{(m)} \right] - \text{Cov}_t (M_{t,t+n}R_{t,t+n}, \mathbb{E}_{t+n} [r_{t+n,t+n+m}])$$

- Covariance term now relates to pricing of *discount-rate risk*, rather than *realized-return risk*
- Likely much smaller than previous term: expected returns are much less volatile than realized returns
- Can be disciplined empirically or theoretically
- Basic idea of proof: $MR_{t,t+n}$ is orthogonal to unexpected component of $r_{t+n,t+n+m}$
The General Case: Result

Define forecast-error proxy and expected-return proxy:

\[
\hat{\varepsilon}_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} - \mathcal{L}_{t}^{(n+m)} + \mathcal{L}_{t}^{(n)}
\]

\[
\hat{\mu}_{t+n}^{(m)} = \mathcal{L}_{t+n}^{(m)} + r_{t+n,t+n+m}^{f}
\]

Result 2 (Generalized Identification)

For any SDF \( M_{t,t+n} \),

\[
\mathbb{E}_t \left[ \hat{\varepsilon}_{t+n}^{(m)} \right] = \mathbb{E}_t \left[ \varepsilon_{t+n}^{(m)} \right] - \text{Cov}_t \left( M_{t,t+n} R_{t,t+n} , \hat{\mu}_{t+n}^{(m)} \right)
\]

- Intuition from log-normal case carries over, with \( \mathbb{E}_{t+n} [r_{t+n,t+n+m}] \) replaced by \( \hat{\mu}_{t+n}^{(m)} \)
- LVIX-based \( \hat{\mu}_{t+n}^{(m)} \) is closely related to \( \mathbb{E}_{t+n} [r_{t+n,t+n+m}] \) but \( \hat{\mu}_{t+n}^{(m)} \) is directly observable
- Main specification: \( \hat{\mu}_{t+n}^{(m)} \) is \( \frac{1}{10} \) as volatile as realized return \( r_{t+n,t+n+m} \)
  \( \implies \) unobserved covariance likely much smaller for forecast errors than for spot rates
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Data and Measurement: Options

Data:

- Main data: Global panel of index options from OptionMetrics (monthly data, standard filters)
  - For U.S. sample: 1990–2021
  - For international sample: Consider 10 major indices, with data since at least 2006
- Sample monthly and apply standard filters
- Baseline: 6-month horizon, 6 months forward ($n = m = 6$)

Measuring LVIX: Following Gao & Martin (2021), Carr & Madan (2001),

$$\mathcal{L}_t^{(n)} = (R_{t,t+n}^f)^{-1} \mathbb{E}_t^*[R_{t,t+n}r_{t,t+n}] - r_{t,t+n}^f = \frac{1}{P_t} \left\{ \int_{F_t^{(n)}}^\infty \frac{\text{put}_t^{(n)}(K)}{K} dK + \int_{F_t^{(n)}}^\infty \frac{\text{call}_t^{(n)}(K)}{K} dK \right\}$$

- Calculate integral a bunch of different ways
- First: Simplify by working under log assumption, so LVIX $\implies$ spot & forward rates
Estimates: Contemporaneous U.S. Spot and Forward Rates

![Graph showing annualized percent of current spot rate and forward rate over time from 1990 to 2020. The graph has shaded areas highlighting specific periods.]
Estimates: Realized U.S. Spot and Forward Rates

- Realized Spot Rate
- Forward Rate

Forecast Error
### Mincer–Zarnowitz Regressions for Spot Rates by Country

<table>
<thead>
<tr>
<th></th>
<th>(1) U.S.</th>
<th>(2) Ex-U.S.</th>
<th>(3) All</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_t^{(6,6)}$</td>
<td>$0.67^{***}$</td>
<td>$0.55^{***}$</td>
<td>$0.56^{***}$</td>
</tr>
<tr>
<td></td>
<td>$(0.096)$</td>
<td>$(0.056)$</td>
<td>$(0.055)$</td>
</tr>
<tr>
<td>Intercept</td>
<td>$0.74^{***}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(0.28)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Country FEs</td>
<td>$\times$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>p-value: $\beta_1 = 1$</td>
<td>$0.003$</td>
<td>$0.000$</td>
<td>$0.000$</td>
</tr>
<tr>
<td>Obs.</td>
<td>378</td>
<td>1,849</td>
<td>2,227</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.22</td>
<td>0.21</td>
<td>0.22</td>
</tr>
<tr>
<td>Within $R^2$</td>
<td>—</td>
<td>0.14</td>
<td>0.15</td>
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SEs in (1) are heteroskedasticity and autocorrelation-robust [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Substantial predictive power...
- ...but $\beta_1 \neq 1$, suggesting forward rates overshoot future spot rates
- What if $\beta_1$ estimate is downwardly biased due to measurement error?
- To address this, now consider IV using shorter-term forward rate $f_t^{(2,1)}$ as instrument for $f_t^{(6,6)}$
- Shorter-horizon forwards likely to be better measured: denser option strikes & more trading volume
### Instrumented Mincer–Zarnowitz Regressions for Spot Rates

\[
\mu_{t+6}^{(6)} = \beta_0 + \beta_1 f_t^{(6,6)} + \epsilon_{t+6}, \quad f_t^{(6,6)} = \pi_0 + \pi_1 f_t^{(2,1)} + \eta_t
\]

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<tr>
<td></td>
<td>U.S.</td>
<td>Ex-U.S.</td>
<td>All</td>
</tr>
<tr>
<td>(f_t^{(6,6)})</td>
<td>0.73***</td>
<td>0.69***</td>
<td>0.70***</td>
</tr>
<tr>
<td>(0.062)</td>
<td>(0.078)</td>
<td>(0.074)</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>0.59***</td>
<td>(0.13)</td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>✗</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>(p)-value: (\beta_1 = 1)</td>
<td>0.018</td>
<td>0.004</td>
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- Forward rate \(\uparrow\) by 1% \(\implies\) future spot rate \(\uparrow\) by \(\sim 0.7\%\)
- Forward rates explain \(\sim 20\%\) of the variation in future spot rates
- The market qualitatively understands variation in the equity premium, but quantitatively significant excess persistence

SEs in (1) are heteroskedasticity and autocorrelation-robust [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.
**Average Forecast Errors Across Countries**

\[ \varepsilon_{t+6}^{(6)} = \mu_{t+6}^{(6)} - f_{t}^{(6,6)} \]

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<tr>
<td>Average</td>
<td>0.021</td>
<td>0.20</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.11)</td>
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SEs in (1) are HAR [Lazarus et al. (2018)], and in (2)–(3) are clustered by exchange and month.

- Not just statistically insignificant, but effectively zero: \( \bar{\varepsilon} \leq 20 \) bps annualized
- Therefore can’t reject log utility + RE just on the basis of average errors
  - Not the highest-powered test, but will be informative in trying to rationalize time variation
- But average of zero masks substantial predictability…
Forecast Errors and Lagged Forward Rates Over Time

The diagram illustrates the annualized percent changes over time from 1990 to 2020, highlighting various economic events such as the Russian Financial Crisis, Stock Market Downturn of 2002, Global Financial Crisis, Double-Dip Recession Fears, Debt-Ceiling Crisis, and Covid-19 Recession.
Forward Rates as Predictors of Forecast Errors

**Forward Rates (IV)**

**Predicted Forecast Errors**
Predictable Forecast Errors

Regressions of Forecast Errors on $2 \times 1$ Forward Rate

$$\varepsilon_{t+6}^{(6)} = \beta_0 + \beta_1 f_{t}^{(2,1)} + e_{t+6}$$

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- Forward rates again overshoot future spot rates
- Errors are also predictable in Coibion–Gorodnichenko regressions using forward-rate revisions
- And predictability rises substantially ($R^2 = 0.11$) with kernel regression: Arises mostly from high forward rates
- Is this consistent with overreaction? It depends: Overreaction to what?
  - Option-based expected returns: Yes
  [Spot rates, fwd rates, fwd-rate revisions]
  - Past returns: Wrong direction
  - Consistent excess persistence
How Significant Are Forecast Errors?

Can now return to question posed at outset:

How significant are forecast errors for price variation?

\[ p_t - d_t = \kappa - \sum_{j=0}^{\infty} \rho^j E_t r_{t+j+1} + \sum_{j=0}^{\infty} \rho^j E_t \Delta d_{t+j+1} \]
How Significant Are Forecast Errors?

Can now return to question posed at outset:

**How significant are forecast errors for price variation?**

\[
pt - dt = \kappa - \mathbb{E}_t r_{t+1} - \sum_{j=1}^{\infty} \rho^j f_t^{(j,1)} - RF_t + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t \Delta d_{t+j+1}
\]

Break \( f_t^{(j,1)} \) into:

\[
f_t^{(j,1)} = \mathbb{E}_t [\mu_{t+j}^{(1)}] - \mathbb{E}_t [\epsilon_{t+j}^{(1)}]
\]

- **Set one period to be 6 months, and predict error using 2m×1m forward**
- **Assume** \( \mathbb{E}_t [\epsilon_{t+j+1}^{(1)}] = \phi^j \mathbb{E}_t [\epsilon_{t+j}^{(1)}] \) \([\text{De la O & Myers (2021)}]\) \(\implies \hat{\phi} \approx 1 \) (using longer-dated SX5E data)
- **Use this to estimate contribution of discounted sum of predicted forecast errors (\( \mathcal{E}_t \)) on prices**
- **Compare to repurchase-adj.** \( pt - dt \) from Nagel & Xu (2022)
Discounted Forecast Errors and Price-Dividend Variation

Meaningful in magnitude, esp. in crises, and on average $\mathcal{E}_t$ moves 0.5-for-one with $p_t - d_t$
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Data and Measurement: Surveys

Survey Data:

▶ Livingston survey of prof. forecasters:
  ▶ Price expectations at 6m & 12m horizon, allow for:
    ▶ $\mu_t^{(12 \text{ months})}, \mu_t^{(6 \text{ months})}$
    ▶ $f_t^{(6 \text{ months})} = \mu_t^{(12 \text{ months})} - \mu_t^{(6 \text{ months})}$
    ▶ $\epsilon_t^{(6 \text{ months})} = \mu_t^{(6 \text{ months})} - f_t^{(6 \text{ months})}$

▶ Duke CFO survey:
  ▶ 1y & 10y return expectations, allow for:
    ▶ $\mu_t^{(10 \text{ years})}, \mu_t^{(1 \text{ year})}$
    ▶ $f_t^{(9 \text{ years}, 1 \text{ year})} = \mu_t^{(10 \text{ years})} - \mu_t^{(1 \text{ year})}$
    ▶ $\epsilon_t^{(9 \text{ years})} \approx \mu_t^{(10 \text{ years})} \times 9/10 - f_t^{(9 \text{ years}, 1 \text{ year})}$
Reminder: Forward Rates and Predicted Forecast Errors

FORWARD RATES (IV)

PREDICTED FORECAST ERRORS
Excess Persistence: Consistent Evidence in Surveys

**LIVINGSTON SURVEY: 6M6M FORWARD RATES**

**PREDICTED FORECAST ERRORS**

Annualized Percent

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>-10</td>
<td>10</td>
<td>0</td>
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<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
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<tbody>
<tr>
<td>Value</td>
<td>-2</td>
<td>2</td>
<td>0</td>
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</table>

24
Excess Persistence: Consistent Evidence in Surveys

CFO Survey: 1Y9Y Forward Rates

Forecast Errors
Predictive Regressions in Survey Data

<table>
<thead>
<tr>
<th>Expectations Measured by:</th>
<th>Livingston Options Survey</th>
<th>CFO Survey</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A. Predictability in Spot Rates ($\mu_{i,t+1} = \beta_0 + \beta_1 f_{i,t} + e_{i,t+1}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.88</td>
<td>0.68</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.71</td>
<td>0.38</td>
</tr>
<tr>
<td>Panel B. Predictability of Forecast Errors ($e_{i,t+1} = \beta_0 + \beta_1 f_{i,t} + e_{i,t+1}$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>-0.34</td>
<td>-0.19</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.06</td>
<td>0.06</td>
</tr>
</tbody>
</table>
### Consistency Across Measures

#### Panel A. Correlation in Forward Rates Across Measures

<table>
<thead>
<tr>
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<th>Expectations Measured by:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Options</td>
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<tr>
<td>Options</td>
<td>1</td>
</tr>
<tr>
<td>Livingston Survey</td>
<td>1</td>
</tr>
</tbody>
</table>

#### Panel B. Cyclical Variation in Forward Rates and Forecast Errors

\[
\rho (f_{i,t}, 1/\text{CAPE}_t) \quad 0.04 & 0.42 & 0.21 \\
\rho (\epsilon_{i,t+1}, 1/\text{CAPE}_t) \quad -0.38 & -0.19 & -0.38 
\]
CFO Spot and Forward Rates: Importance of Long Horizon

Annualized Percent

1y9y Forward
1y Spot

2005 2010 2015 2020
Roadmap

1. Introduction

2. Price-Based Measurement of Expectations: Theory

3. Evidence from Price-Based Expectations

4. Evidence from Survey-Based Expectations

5. Explaining Forecast Errors
   Can Forecast Errors From Price-Based Measure Be Rationalized?
   A Model of Expectation Errors

6. Implications and Conclusions
Explaining Forecast Errors

Paper considers two alternatives explaining forecast errors:

1. RE + risk premium
   - Price of discount-rate risk must be highly volatile and countercyclical for this to work
   - E.g., if $\text{Corr}_t(r_{t+1}, E_{t+1}r_{t+2}) = -1$ and negative correlation condition (Gao & Martin 2021) $\implies$ relevant SDF-related covariance can’t change sign
   - Doesn’t work for survey evidence

2. Expectation errors
   - Simple calibrated model with log util. & diagnostic expectations
   - Increase in equity premium $\implies$ investors overestimate future equity premium
   - Single parameter $\theta$ governs overreaction to objective news
   - Consider range of values, incl. $\theta = 0$ [RE] & $\theta = 0.91$ [Bordalo et al. (2018, 2019) estimate]
   - $\theta$ around 0.9 does well at generating model coefficients close to our empirical estimates
Can Forecast Errors From Price-Based Measure Be Rationalized?

\[ \mathbb{E}_t[\hat{\varepsilon}_{t+n}^{(m)}] = \mathbb{E}_t[\varepsilon_{t+n}^{(m)}] - \underbrace{\text{Cov}_t(MR_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}])}_{\xi_t} \]

What conditions do we need on \( \xi_t \) in order for expectation errors \( \mathbb{E}_t[\varepsilon_{t+n}^{(m)}] \) to be unpredictable?
Must have \(-\xi_t\) take same sign as pred. forecast errors:

Main challenge: Small on average, but must flip signs dramatically (– in good times, + in bad).
Can Forecast Errors From Price-Based Measure Be Rationalized?

\[ E_t[\varepsilon_{t+n}^{(m)}] = E_t[\varepsilon_{t+n}^{(m)}] - \text{Cov}_t(MR_{t,t+n}, E_{t+n}[r_{t+n,t+n+m}]) \]

What conditions do we need on \( \zeta_t \) in order for expectation errors \( E_t[\varepsilon_{t+n}^{(m)}] \) to be unpredictable?

► For simplicity: Take \( n = m = 1 \), and assume SDF and returns are jointly log-normal

► Then \( \zeta_t > 0 \) (as needed in bad times) if and only if:

\[ SR_t(-\mu_{t+1}) > -\rho_t(r, \mu)\sigma_t(r), \]

where \( SR_t \) is Sharpe ratio on claim to next period’s negative equity premium (low payoff is bad)

► Correlation \( \rho_t(r, \mu) \) likely to be negative; for illustration, set it to \(-1\)

► Then \( SR_t \) must vary more than \( \sigma_t(r) \) for \( \zeta_t \) to flip signs

► One calibration: Go back to log utility (likely to be conservative for time variation in \( \sigma_t \)), and estimate \( \sigma_t(r) \) from options
Can Forecast Errors From Price-Based Measure Be Rationalized?

\[ \mathbb{E}_t[\hat{\varepsilon}_{t+n}^{(m)}] = \mathbb{E}_t[\varepsilon_{t+n}^{(m)}] - \text{Cov}_t(MR_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}]) \]

What conditions do we need on \( \varsigma_t \) in order for expectation errors \( \mathbb{E}_t[\varepsilon_{t+n}^{(m)}] \) to be unpredictable?

- \( SR_t \) must vary more than \( \sigma_t(r) \) for \( \varsigma_t \) to flip signs
- One calibration: Go back to conservative log utility case, and estimate \( \sigma_t(r) \) from options.

Results for conditional volatility of 6-month market return:
Can Forecast Errors From Price-Based Measure Be Rationalized?

\[
\mathbb{E}_t[\hat{e}_{t+n}^{(m)}] = \mathbb{E}_t[e_{t+n}^{(m)}] - \text{Cov}_t(MR_{t,t+n}, \mathbb{E}_{t+n}[r_{t+n,t+n+m}])
\]

What conditions do we need on \( \zeta_t \) in order for **expectation errors** \( \mathbb{E}_t[\hat{e}_{t+n}^{(m)}] \) to be unpredictable?

- **SR**\(_t\) must vary *more than* \( \sigma_t(r) \) for \( \zeta_t \) to flip signs
- Further, given \( \rho_t(r, \mu) = -1 \), \( \zeta_t \) **cannot** flip signs if mNCC [Gao & Martin (2021), Assumption 2] holds
  - \( \rho_t(r, \mu) = -1 \implies \zeta_t \) is scaled version of their covariance term \( C_t^{(n)} \)
  - If \( C_t^{(n)} \leq 0 \) (mNCC), then \( \zeta_t \leq 0 \)
- More generally, difficult to get both average errors (small) *and* time variation (large) right
- Paper has one illustration varying risk aversion \( \gamma \)
Model Setup

- Now want a simple lab to examine whether findings could plausibly arise from combo of:
  1. Log utility
  2. Expectation errors

⇒ consider a version of framework from Bordalo, Gennaioli, Shleifer (2018)

- 3-month spot rate dynamics under objective measure:

$$
\mu_t^{(3)} = \left(1 - \sum_{j=1}^{3} \phi_j\right) \bar{\mu} + \phi_1 \mu_{t-1}^{(3)} + \phi_2 \mu_{t-2}^{(3)} + \phi_3 \mu_{t-3}^{(3)} + \epsilon_t, \quad \epsilon_t \text{ i.i.d.} \sim \mathcal{N}(0, \sigma^2_{\epsilon})
$$

- Under RE: Term structure of current spot rates would be based on objective expectations $\mathbb{E}_t [\mu_t^{(3)}]$

- Actual subjective expectations: Excess sensitivity to news governed by “diagnosticity” parameter $\theta$:

$$
\mathbb{E}_t^{\theta} [\mu_t^{(3)}] = \mathbb{E}_t [\mu_t^{(3)}] + \theta \left( \mathbb{E}_t [\mu_{t+n}^{(3)}] - \mathbb{E}_{t-3} [\mu_{t+n}^{(3)}] \right) \text{ news } \propto \epsilon_t
$$
Model Setup

\[ \mu_t^{(3)} = \left( 1 - \sum_{j=1}^{3} \phi_j \right) \bar{\mu} + \phi_1 \mu_{t-1}^{(3)} + \phi_2 \mu_{t-2}^{(3)} + \phi_3 \mu_{t-3}^{(3)} + \epsilon_t, \quad \epsilon_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2_\epsilon) \]

\[ \mathbb{E}_t^{\theta} \left[ \mu_{t+n}^{(3)} \right] = \mathbb{E}_t \left[ \mu_{t+n}^{(3)} \right] + \theta \left( \mathbb{E}_t \left[ \mu_{t+n}^{(3)} \right] - \mathbb{E}_{t-3} \left[ \mu_{t+n}^{(3)} \right] \right) \]

- Forward rates based on subjective expectations
- Longer-term spot rates embed objective short rate and subjective expectations of future short rates
- Consider a range of values for \( \theta \)
  - \( \theta = 0 \): RE
  - \( \theta = 0.91 \): BGS (2018), BGLS (2019)
- Estimate objective parameters for spot-rate process in each country
- For each \( \theta \), simulate 10,000 samples and run same tests as in the data for \( n, m = 6 \) months
Model vs. Data: Main Estimates

Mincer–Zarnowitz Regressions

Predictability of Forecast Errors

Average Forecast Errors

Coibion–Gorodnichenko Regressions
Model vs. Data: $R^2$ Values

Simple calibration does reasonably well on main estimates…

…but seems to miss some rational variation in forward rates:

![Graph showing Mincer–Zarnowitz Regressions](image)

![Graph showing Coibion–Gorodnichenko Regressions](image)
A Trilemma for Expectation Errors

More generally:

- While simple calibrated model does reasonably well at matching the data, again not an unqualified success for all possible notions of overreaction

- Subjective beliefs overreact to increases in *spot rates* in our model, not past returns, and cyclicality matters:

$$p_t - d_t = \kappa - \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t r_{t+j+1} \mathbb{E}_t r_{t+1} - \sum_{j=1}^{\infty} \rho^j f_t^{(j,1)} + \sum_{j=0}^{\infty} \rho^j \mathbb{E}_t \Delta d_{t+j+1} - RF_t$$

- Use \(\sim\) to denote **expectation error wedge** (deviation from RE economy):

$$\text{var}(\widetilde{p_t - d_t}) = \text{var}(\widetilde{F_t}) + \text{var}(\widetilde{CF_t}) - 2 \text{cov}(\widetilde{F_t}, \widetilde{CF_t})$$

- Have to choose between **two of three**:
  1. Volatile expectation errors for cash flows and/or returns
  2. Volatile price-dividend ratio relative to RE
  3. Positive comovement between fundamental and return expectation errors
Roadmap

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Implications

Excess persistence in expected returns helps us understand:

1. Excess volatility in stock prices
   - When prices are depressed, this partly reflects investors expecting persistently high risk premia
   - If investors didn’t overestimate persistence, would see more modest fluctuations in prices (about 50% less during 2008 crisis, nearly 100% less during Covid crash)

2. Inelastic demand for equities [Gabaix & Koijen 2022]
   - Puzzle: Why investors change weight in equities so modestly in response to change in stock prices
   - Partial resolution: If price drop leads to increases in expected returns mainly at long horizons, shouldn’t see big increase in portfolio weight
Equity premium forecast errors help explain excess volatility, especially in crises.
Implications

Excess persistence in expected returns helps us understand:

3. Facts about equity term structure from dividend claims
   ▶ Risk premia lower than expected $\Rightarrow$ Realized returns higher than expected
   ▶ Effect stronger for longer-duration assets (the market)
   ▶ Potentially explains:
     ▶ Downward sloping equity term structure on average [Binsbergen, Brandt, Koijen 2012]
     ▶ Upward sloping term structure during bad times (counter-cyclical variation) [Gormsen 2021]

4. Debate on cyclicality of subjective risk premia
   ▶ Short-term return expectations sometimes appear procyclical [Greenwood & Shleifer 2014], acyclical [Nagel & Xu 2023], or countercyclical [Dahlquist & Ibert 2022]
   ▶ Forward expectations are countercyclical across all data sources
   ▶ Disagreement in above studies may stem in part from differences in horizon
Final Notes

Summary:
▶ Introduce new methods to measure term structure of expected equity premia
▶ Robust evidence of excess persistence in return expectations
▶ Investors consistently overestimate how long their own expected returns will stay elevated during bad times, and vice versa during good times
▶ Consistent across options (high-powered, general method) and surveys (straightforward measurement)

Tie-ins:
▶ Equity and bond term structures: Our tests are similar to tests of the expectations hypothesis, but with less room for discount-rate variation than in past work
▶ Similar to past work [van Binsbergen & Koijen (2017), Gormsen (2021)], find more predictability in equity term structure than in FI term structure
▶ Also build on Giglio & Kelly (2018) work on other term structures